Homework 2 – due 02/13/04

Math 341

Recommended problems:

Problems to be handed in:

1. Find a parametrization for the curve Γ and evaluate the line integral of the function $f : \mathbb{R}^3 \to \mathbb{R}$, where $f(x, y, z) = xyz$ and

(a) ImΓ is the polygon whose successive vertices are $(0, 0, 1), (0, 1, 1)$, and $(1, 2, 3)$.

(b) ImΓ is the unit circle in the plane $z = 1$ with center $(1, 1, 1)$.

2. Evaluate the line integral of the vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ given by $\mathbf{F}(x, y, z) = (2x, z, y)$ over the line Γ defined by $\alpha(t) = (t, t^2, t^3)$. (This line is called the *twisted cubic.*)

3. (a) Show that the relation "α is equivalent to β" is an equivalence relation.
(b) Show that the relation "α is o-equivalent to β" is an equivalence relation.

4. (a) Find a potential function for each of the following force fields and determine the work required to move a particle from the point $\mathbf{v}_0$ to $\mathbf{v}_1$. (The work done is $-\int_\Gamma \mathbf{F}$ since one pushes against the force.)

(i) $\mathbf{F}(x, y, z) = (2xy, x^2 + z, y)$, $\mathbf{v}_0 = (1, 3, 2)$, $\mathbf{v}_1 = (2, 3, 1)$.

(ii) $\mathbf{F}((x, y, z) = (yz\cos(xyz), xz\cos(xyz), xy\cos(xyz))$, $\mathbf{v}_0 = (1, 1, 0)$, $\mathbf{v}_1 = (1/3, 1/2, 2\pi)$.

(b) Show that if $\mathbf{F} = (F_1, \ldots, F_n)$ is a conservative vector field on $\mathbb{R}^n$, then

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_j}{\partial x_i}$$

on $\mathbb{R}^n$, for all i, j.

5. We say $\alpha : [a, b] \to \mathbb{R}^n$, a parametrization of Γ, is a *parametrization by arc-length* if for all $t_0 \in [a, b]$, the length of the piece of α between $t = a$ and $t = t_0$ is exactly $t_0 - a$. Show that α is a parametrization by arc-length if and only if

$$|\alpha'(t)| = 1$$

for all t. (Remember that we are assuming $\alpha'(t)$ is a continuous function of t.)