Practice 1 – For exam 02/27/04

Math 341

Recommended problems:

1. (a) Let $\mathbf{F}$ be a C^2-vector field on an open region $U \subset \mathbb{R}^3$. Show that $\operatorname{div}\operatorname{curl}\mathbf{F} = 0$.
(b) Let $f : U \to \mathbb{R}$ be a C^2-function (U as above). Show that $\operatorname{curl}\operatorname{grad} f = \mathbf{0}$.

2. Let S be the surface of the sphere $(x-2)^2 + (y-3)^2 + (z-1)^2 = 25$. Evaluate

$$\int_S x^2\,dy\,dz + y^2\,dz\,dx + z^2\,dx\,dy.$$

3. Let D be a closed and bounded region in $\mathbb{R}^3$, and suppose that Gauss' theorem can be applied to D and ∂D. Show that the volume of D is given by

$$\operatorname{Vol}(D) = \int_{\partial D} x\,dy\,dz = \int_{\partial D} y\,dz\,dx = \int_{\partial D} z\,dx\,dy.$$

4. Prove Green's identities: If f and $g : \mathbb{R}^3 \to \mathbb{R}$ are C^2-functions in the region D (as above), then

$$\int_D (f\Delta g + (\nabla f \cdot \nabla g)) = \int_{\partial D} f\nabla g$$

and

$$\int_D (f\Delta g - g\Delta f) = \int_{\partial D} (f\nabla g - g\nabla f).$$

Here

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

is the Lapacian of f, and $\nabla f = \operatorname{grad} f$. (Hint: Let $\mathbf{F} = f\nabla g$ in Gauss' theorem.)

5. (a) Let C be any closed and bounded region of $\mathbb{R}^3$ which does not contain the origin, and let $\mathbf{F}(\mathbf{v}) = \dfrac{\mathbf{v}}{|\mathbf{v}|^3}$. Show that

$$\int_{\partial C} \mathbf{F} = 0.$$

(b) Let V be a ball in $\mathbb{R}^3$ centered at the origin, and let $\mathbf{F}$ be as in (a). Show that

$$\int_{\partial V} \mathbf{F} = 4\pi.$$