Exam I – 10/05/06

Math 406

Name:

1. (a) (10 points) Find the prime factorization of 300093885.

(b) (15 points) The number 1044541 is the product of two prime numbers. Find them.

2. (a) (10 points) Find all solutions to the following congruence equation:

$$28x \equiv 4 \pmod{34}.$$

(b) (15 points) How many different ways can you pay 84 cents in postage, using only 7 cent and 15 cent stamps?

3. (15 points) Let n be an integer, $n \geq 2$. Show that the last digit in the decimal expansion of 2^{2^n} is 6. Hint: Consider the numbers modulo 10, and use induction.

4. (10 points) Find the least positive residue of 2^{51} mod 15.

5. (10 points) Let p denote a prime number. Recall that the notation $p^c \| z$ means that p^c is the highest power of p which divides z. Suppose that $p^a \| x$ and $p^b \| y$. Show that $p^{\min(a,b)} \| (x+y)$.

6. (15 points) Find (1011,96) and write it as a linear combination of the two numbers.