Homework 6 – due 10/10/07

Math 600

25. (5 points) Suppose $1 \to A \to B \to C \to 1$ is an exact sequence of groups. Prove that $|A|$ and $|C|$ are finite if and only if $|B|$ is finite, in which case $|B| = |A| \cdot |C|$.

26. (10 points) Suppose G is a group with $|G| = 5 \cdot 11 \cdot 17$. Suppose that G has an element of order 55. Show that G is cyclic.

27. (10 points) Suppose the semidirect product $G \rtimes \mathbb{Z}$ is such that the action of $1 \in \mathbb{Z}$ is an inner automorphism $\mathrm{Int}(g)$ of G. Show that $G \rtimes \mathbb{Z} \cong G \times \mathbb{Z}$ (and find an explicit isomorphism). HINT: this problem is a special case of Dummit-Foote, 5.5, #6.

28. (10 points) Dummit-Foote, 5.5, #18.

29. (10 points) Dummit-Foote, 6.1, #12.