Calculus 140, section 5.5 Indefinite Integrals and Integration Rules

notes by Tim Pilachowski

4.3 Example A redux: Given a function $f(x) = 5x^4$ find a function *F*(*x*) such that $F'(x) = f(x)$.

answer: $F(x) = x^5 + C$

In the example above, the question was phrased, "Find a function *F*(*x*) such that $F'(x) = f(x)$." There are four other equivalent ways to ask the same thing:

Find all antiderivatives of *f*(*x*).	Integrate *f*(*x*).
Find the integral of *f*(*x*).	Find $\int f(x)\,dx$.

These are called the **indefinite integral of *f*** [Definition 5.15].

Example B: Find all antiderivatives of $f(x) = x^4$. *answer*: $\frac{1}{5}x^5 + C$

From this example, we can generalize the process for integrating power functions:

$$\int x^r \, dx = \frac{1}{r+1} x^{r+1} + C, \quad r \neq -1.$$

Note the restriction on *r*. We have to avoid a 0 in the denominator since division by 0 is undefined.

We'll take a close look at $\int \frac{1}{x}\,dx$ in a later section. Until that time, we'll assume we are integrating $\frac{1}{x}$ only for positive values of *x*.

Example C: Evaluate $\int \frac{1}{\sqrt{x}}\,dx$. *answer*: $2\sqrt{x} + C$

Now is as good a time as any to point out the "dx" part of the integral $\int f(x)\,dx$. It is a necessary part of any integral, since we are finding the antiderivative "with respect to x: $f(x)=\frac{d}{dx}[F(x)] \;\Leftrightarrow\; \int f(x)\,dx = F(x)+C$.

Example D: Evaluate $\int e\,dx$. *answer*: $ex + C$

Note that in this example, as in all the others, we can easily check our answer by finding its derivative: $\frac{d}{dx}(e\,x+C)=e$, which is correct.
Checking your integration by finding the derivative is a good habit to develop.

IMPORTANT NOTE:

Just like differentiation, integration has a sum rule [Theorem 5.16 and Corollary 5.18].

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}[f(x)]+\frac{d}{dx}[g(x)] \;\Rightarrow$$

$$\int[f(x)+g(x)]\,dx=\int f(x)\,dx+\int g(x)\,dx \qquad \int_a^b[f(x)+g(x)]\,dx=\int_a^b f(x)\,dx+\int_a^b g(x)\,dx$$

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}[f(x)]-\frac{d}{dx}[g(x)] \;\Rightarrow$$

$$\int[f(x)-g(x)]\,dx=\int f(x)\,dx-\int g(x)\,dx \qquad \int_a^b[f(x)-g(x)]\,dx=\int_a^b f(x)\,dx-\int_a^b g(x)\,dx$$

Example E: Find $\int(\cos t-\sin t)\,dt$. *answer*: $\sin t+\cos t+C$

Just like differentiation, integration has a constant multiple rule [Theorem 5.17].

$$\frac{d}{dx}[c*f(x)]=c*\frac{d}{dx}[f(x)] \;\Rightarrow$$

$$\int c*f(x)\,dx=c*\int f(x)\,dx \qquad \int_a^b c*f(x)\,dx=c*\int_a^b f(x)\,dx$$

Example F: Find the integral of $f(x)=\frac{7}{9}e^{x}$. *answer*: $\frac{7}{9}e^{x}+C$

Example G: Evaluate $\int_1^4 \left(x-\sqrt{x}\right)^2 \, dx$. *answer*: $\frac{111}{30}$

Example H: Evaluate $\int_1^{\pi/2} \left(\sqrt[3]{x^2} + \frac{1}{4x} - 5e^{x} + 6\sin x + 7\cos x \right) dx$.

answer: $\frac{3}{5}\left(\frac{\pi}{2}\right)^{5/3} + \frac{1}{4}\ln\left(\frac{\pi}{2}\right) - 5e^{\pi/2} + 5e + 6\cos(1) - 7\sin(1) + \frac{32}{5}$

(Note that domain of $\ln x$ is not an issue, since the interval I over which we're integrating contains only positive values for x.)