.

GEOMETRIC STRUCTURES ON MANIFOLDS: CORRECTIONS

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1. Radiance

The definition of a *radiant vector field* in Definition 1.6.3 (p.15) needs to be stronger. A radiant vector field is required to be normalized so that the scale factor for the time-t map of the corresponding flow equals e^t.

A related misprint occurs on p.16, Exercise 1.6.5, second bullet. A vector field X is radiant $\iff$ the covariant derivative $\nabla_Y(X) = Y$ for all vector fields Y. (It is incorrectly stated that $\nabla_Y(X) = X$.)

2. Convergent sequences of collineations

The second sentence of the proof of Lemma 2.6.2, on p.43, needs clarification. The lemma concerns two vector spaces V, V' and a sequence of linear maps $\mathsf{V} \xrightarrow{f_n} \mathsf{V}'$ converging to a linear map f_∞. The lemma asserts that for given a compact subset $\widetilde{K}$ of the complement $\mathsf{V} \setminus \mathsf{Ker}(f_\infty)$, and a fixed $\epsilon > 0$, there exists N such that $n > N$, then $\forall x \in \widetilde{K}$,

$$\|\widetilde{f}_\infty(x) - \widetilde{f}_n(x)\| < \frac{C\epsilon}{2},$$

$$\left|1 - \frac{\widetilde{f}_n(x)}{\|\widetilde{f}_\infty(x)\|}\right| < \frac{\epsilon}{2}. \tag{1}$$

There is a misprint in the second inequality, which should read:

$$\left|1 - \frac{\|\widetilde{f}_n(x)\|}{\|\widetilde{f}_\infty(x)\|}\right| < \frac{\epsilon}{2} \tag{2}$$

since without the norm signs in the numerator, it makes no sense.

That such an N exists, independent of x, can be seen as follows. First, recall that $\epsilon, C, f_n, f_\infty$ are given. Denote by $W(x, n)$ the set of all $x \in \widetilde{K}$ satisfying (1) and (2).

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Clearly $W(x,n)$ is an open neighborhood of x. Since the $\widetilde{f_n}$ converge pointwise to $\widetilde{f_\infty}$, there exists $N(x)$ such that $W(x,n)$ is an open neighborhood of x for $n \geq N(x)$.

Furthermore (1) and (2), describes an open subset of x. If $y \in \widetilde{K}$, then $W(y) := W\big(y, N(y)\big)$ is an open neighborhood $W(y)$ of y. Moreover (1) and (2) holds for $n > N(y)$ and $x \in W(y)$.

Since $\widetilde{K}$ is compact, $\exists y_1, \dots, y_l \in \widetilde{K}$ such that

$$\widetilde{K} \subset W(y_1) \cup \dots W(y_l).$$

Let $N := \max\big(N(y_1), \dots N(y_k)\big)$. Then for every $n > N$, *every* $x \in \widetilde{K}$ satisfies (1) and (2) as desired.

(I am grateful to Sean Lawton for raising this point.)

3. Exponential map for $\mathsf{Aff}_+(1,\mathbb{R})$

§10.9, contains a misprint (bottom of p.234), for the translational part of the exponential map for the group $\mathsf{Aff}_+(1,\mathbb{R})$. The correct formula is: The exponential map is:

$$\begin{aligned} \mathsf{aff}(1,\mathbb{R}) &\xrightarrow{\exp} \mathsf{Aff}_+(1,\mathbb{R}) \\ \big[y \mid x\big] &\longmapsto \big[e^y \mid \varepsilon_y(x)\big] \end{aligned}$$

where ε_y denotes the continuous function $\mathbb{R} \to \mathbb{R}$ defined by:

(3) $$\varepsilon_y(x) := \begin{cases} \frac{e^y - 1}{y}\, x & \text{if } y \neq 0 \\ x & \text{if } y = 0 \end{cases}$$

instead of:

The exponential map is:

$$\begin{aligned} \mathsf{aff}(1,\mathbb{R}) &\xrightarrow{\exp} \mathsf{Aff}_+(1,\mathbb{R}) \\ \big[y \mid x\big] &\longmapsto \big[e^y \mid \varepsilon_y(x)\big] \end{aligned}$$

where ε_y denotes the continuous function $\mathbb{R} \to \mathbb{R}$ defined by:

$$\varepsilon_y(t) := \begin{cases} \frac{e^{yt} - 1}{y} & \text{if } y \neq 0 \\ t & \text{if } y = 0 \end{cases}$$