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GEOMETRIC STRUCTURES ON MANIFOLDS: CORRECTIONS

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I'd like to thank Toshiki Fujii of Osaka University for the following corrections.

1. Typos

1.1. **Riemannian structure on S^2.** On p. 55, Exercise 3.2.7, the expression for the metric tensor should have $(x^2+y^2+1)^2$ in the denominator, not just $(x^2+y^2+1)^2$. The correct formula is:

$$\frac{dx^2+dy^2+(xdy-ydx)^2}{(x^2+y^2+1)^2}$$

1.2. **Number of affine Lie group structures on $\mathbb{R}^2$.** On p.121, Exercise 5.5.9, there are **six** structures on the Lie group $G \cong \mathbb{R}^2$, not five.

1.3. **Equivalence of marked structures.** On p.162, Definition 7.1.3 there is a typo:

"Say that two marked (G,X)-manifolds (M,f) and (M',f') are equivalent if and only if a (G,X)-isomorphism $M \xrightarrow{\phi} M'$ exists such that $\phi \circ f$ is diffeotopic to f'."

The last "f'" was originally incorrectly "ϕ," which makes no sense.

2. Grafting of $\mathbb{R}\mathsf{P}^1$-manifolds

On p.122, Exercise 5.5.11, the fourth bullet item should be revised: "M is obtained by grafting a homogeneous (affine) 1-manifold with some positive number of copies of the model $\mathbb{R}\mathsf{P}^1$-manifold M_0 (given by an isomorphism $M_0 \cong \mathbb{R}\mathsf{P}^1$."

Alternatively we could replace M_0 by its n-fold cover $n \geq 1$ which is obtained by grafting n copies of M_0.

In general if M is a compact $\mathbb{R}\mathsf{P}^1$-manifold with parabolic holonomy Γ, the subset M_∞ of points in M which develop to the fixed point of Γ is a finite set. Let $n \geq 0$ denote its cardinality. If $n = 0$, then M comes from an affine (homogeneous) structure, and Γ is a cyclic

Date: October 24, 2025.

group of unipotent maps, and $M \cong \mathbb{R}/\mathbb{Z}$. If $n > 0$, choose a point $p \in M_\infty$. Then p has a neigborhood N (homeomorphic to an open interval) which is projectively equivalent to an affine line in $\mathbb{R}\mathsf{P}^1$; writing $\mathbb{R}\mathsf{P}^1$ as $\mathbb{R} \cup \{\infty\}$, the affine line N is just $\mathbb{R}$.

Let M_1 denote the closure of the complement $M \setminus N$; it again has parabolic holonomy but $(M_1)_\infty$ has cardinality $n-1$. Furthermore M arises as an identification space of the disjoint union of the closure $\overline{N}$ of N and M_1, which equals the grafting of M_1 and $\mathbb{R}\mathsf{P}^1$. Proceeding by downwards induction on n, one sees that M is obtained by grafting $\mathbb{R}/\mathbb{Z}$ with n-copies of the model structure $\mathbb{R}\mathsf{P}^1$. If $n > 0$, then this is the same as grafting $\mathbb{R}/Z$with the n-fold covering space of $M_0 = \mathbb{R}\mathsf{P}^1$.

3. Confusion in § 6 about the embedding $\mathsf{H}^2 \hookrightarrow \mathbb{C}\mathsf{P}^1$

The statement on p.131 before Theorem 6.1.2 is confusing. Furthermore, Theorem 6.1.2 is incorrectly stated that "every surface admits a $\mathbb{C}\mathsf{P}^1$-structure." Since every $\mathbb{C}\mathsf{P}^1$-structure implies an orientation, we need to assume that the surface is orientable:

"every orientable surface admits a $\mathbb{C}\mathsf{P}^1$-structure."

In the preceding discussion of the embedding of H^2 in $\mathbb{C}\mathsf{P}^1$, we really should use $\mathsf{PSL}(2,\mathbb{R})$ rather than $\mathsf{PGL}(2,\mathbb{R})$, since the orientation-reversing elements of $\mathsf{PGL}(2,\mathbb{R})$ which reverse orientation on H^2 do not preserve the image of H^2. Rather, they take the image of H^2 to its complement in $\mathbb{C}\mathsf{P}^1$. The correct display should be:

$$\big(\mathsf{H}^2, \mathsf{PSL}(2,\mathbb{R})\big) \longrightarrow \big(\mathbb{C}\mathsf{P}^1, \mathsf{PGL}(2,\mathbb{C})\big)$$