MATH 240 Dr. Wolfe SAMPLE EXAM II Fall 2004

1.
   (a) Find $\det C$ where

$$C = \begin{pmatrix} 2 & 0 & 0 & 0 \\ -4 & 1 & 0 & 0 \\ 2 & -2 & -1 & 0 \\ 3 & -1 & 4 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 6 & 5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}.$$

   (b) Suppose (x_1, x_2, x_3) is the solution of the system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 4 \\ 3x_1 + x_2 + 3x_3 &= 7 \\ 4x_1 + 2x_2 + 3x_3 &= 7 \end{aligned}$$

   Express x_2 as a quotient (ratio) of determinants. Do not evaluate the determinants.

2. Let

$$A = \begin{pmatrix} 1 & 2 & 4 & 1 \\ -1 & -1 & 2 & 0 \\ -1 & 1 & 14 & 2 \end{pmatrix}$$

   Find
   (a) the rank of A.
   (b) a basis for the column space of A.
   (c) the dimension of the nullspace of A.
   (d) a basis for the nullspace of A.

3.
   (a) Complete the following definition: A subset S of a vector space V is a *Subspace of* V if ________________
   (b) Let H be the set of all vectors of the form $(s+3t, s-t, 2s-t, 4t)^T$. Show that H is a subspace of $\mathbf{R^4}$, find a basis for H and its dimension.

4. Suppose U is a square matrix such that $UU^T = I$ (i.e. U is orthogonal) Show that $detU = \pm 1$.

5. Let

$$\mathbf{b}_1 = (1,0,0)^T, \quad \mathbf{b}_2 = (2,3,0)^T, \quad \mathbf{b}_3 = (1,2,2)^T, \quad \mathbf{x} = (0,-1,8)^T$$

   (a) Show that the set $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is a basis for $\mathbf{R}^3$.
   (b) Find the change-of-coordinate matrix from $\mathcal{B}$ to the standard basis.
   (c) Write the equation that relates $\mathbf{x}$ in $\mathbf{R}^3$ to $[\mathbf{x}]_{\mathcal{B}}$.
   (d) Find $[\mathbf{x}]_{\mathcal{B}}$ for the $\mathbf{x}$ given above.

6. On any given day, a student is either healthy or ill. Of the students who are healthy today, 95% will be healthy tomorrow. Of the students who are ill today, 55% will still be ill tomorrow.
   (a) Write down the stochastic matrix for this situation.
   (b) In the long run, what is the probability that a student will be healthy on any given day ?