MATH 240 **Third Hour Exam Review** **Fall 2004**

1. Name the only two Mathematics Departments in the USA with all three of the following:
   i A Ph.D Graduate with a Nobel prize.
   ii An undergraduate Math major who went on to win a Fields Medal.
   iii A Fields Medalist on the faculty.

2. Let
$$A = \begin{pmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{pmatrix}$$
is $(1, -2, 1)^T$ an eigenvector of A ? If so, find the eigenvalue.

3. Let
$$B = \begin{pmatrix} 2 & 3 & 3 \\ 12 & 5 & 6 \\ -27 & -15 & -16 \end{pmatrix}$$
Given that $(3 + 2i, 5 - i, -13)^T$ is an eigenvector of B corresponding to the eigenvalue $\lambda = -4 + 3i$, find another eigenvalue of B and a corresponding eigenvector.

4. Let A be a 2×2 matrix whose eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 2$ with corresponding eigenvectors $\mathbf{v_1} = (1, 1)^T$ and $\mathbf{v_2} = (2, 3)^T$ Solve the initial value problem $\mathbf{x}' = A\mathbf{x}, \quad \mathbf{x}(0) = (5, 2)^T$.

5. Let A be as in problem 4. What is A ?

6. Let $\mathbf{u} = (1, 1, 1, 1)^T, \ \mathbf{v} = (1, 7, 1, 7)^T, \ W = \text{Span}\{\mathbf{u}, \mathbf{v}\}$.
   (a) Calculate $\|\mathbf{v}\|$, $\text{dist}(\mathbf{u}, \mathbf{v})$, the projection of $\mathbf{v}$ onto $\mathbf{u}$ and the unit vector in the direction of $\mathbf{u}$.
   (b) Apply the Gram-Schmidt process to $\{\mathbf{u}, \mathbf{v}\}$ to obtain an orthonormal basis for W.
   (c) Let $\mathbf{y} = (3, 2, -1, 2)^T$. Find $\mathbf{z}$, the vector in W which is closest to $\mathbf{y}$.