

1. Describe, in parametric vector form, the solution set of a linear system whose corresponding augmented matrix is row equivalent to

$$\begin{pmatrix} 1 & -2 & 1 & 0 & 6 & -4 \\ 0 & 1 & 3 & 0 & -2 & 2 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

2. Consider the linear system

$$\begin{aligned} x_1 + 4x_2 &= 2 \\ 3x_1 + hx_2 &= k \end{aligned}$$

Determine all values of h and k such that the system has

- (a) no solutions,
- (b) a unique solution,
- (c) many solutions.

3.

- (a) Complete the following definition: Let $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ be a set of vectors in \mathbf{R}^n . The set $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$ is *linearly independent* if _____
- (b) Let

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 3 & 4 & 2 \\ 5 & 2 & 8 \end{pmatrix}.$$

Are the columns of A linearly independent ?

- (c) Define $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ by $T\mathbf{x} = A\mathbf{x}$ (A as in part(b)). Does T map \mathbf{R}^3 onto \mathbf{R}^3 ?
- (d) Let A be as in part (b). Is A invertible ?

4. Let $A = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \end{pmatrix}$. Compute A^{-1} and use A^{-1} to solve the matrix equation $XA = B^T$. (Careful !)

5. Define $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ by $T(x_1, x_2) = (x_1 + x_2, x_1 + 3)$. Is T is a linear transformation ? Explain.

6. A small remote village receives radio broadcasts from two radio stations, a news station and a music station. Of the listeners who are tuned to the news station, 70% will remain listening to the news after the station break which occurs each half hour, while 30% will switch to the music station at the station break. Of the listeners who are tuned to the music station, 60% will switch to the news station at the station break, while 40% will remain listening to the music. Suppose everyone is listening to the news at 8:15 a.m.

- (a) Set up the migration matrix for this situation.

- (b) Give \mathbf{x}_0 , the initial state vector.
 - (c) What percentage of the listeners will be listening to the music station at 9:25 a.m. (after the station breaks at 8:30 a.m. and 9:00 a.m.)?
7. Mark each statement as true (T) or False (F). (No reasons required.)
- (a) If an augmented matrix $[A \ \mathbf{b}]$ is transformed into $[C \ \mathbf{d}]$ by elementary row operations, then the equations $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{d}$ have exactly the same solution set.
 - (b) If an augmented matrix $[A \ \mathbf{b}]$ can be transformed by elementary row operations into reduced row echelon form, then the equation $A\mathbf{x} = \mathbf{b}$ is consistent.
 - (c) If A is an $m \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ is consistent for every \mathbf{b} in \mathbf{R}^m , then A has m pivot columns.
 - (d) If \mathbf{u} , \mathbf{v} , and \mathbf{w} are nonzero vectors in \mathbf{R}^2 , then \mathbf{w} is a linear combination of \mathbf{u} and \mathbf{v} .