MATH 240 Dr. Wolfe SAMPLE SECOND HOUR EXAM FALL 2008

1. Complete the following definitions:

(a) Let T be a mapping from a vector space V to a vector space W. T is a *linear transformation* if ______________

(b) Let A be a matrix. The *rank* of A is ______________

(c) Let $\mathcal{B} = \{\mathbf{b_1}, \ldots, \mathbf{b_n}\}$ be a basis for a vector space V and $\mathbf{x} \in V$. The *$\mathcal{B}$-coordinate vector of* $\mathbf{x}$, $[\mathbf{x}]_\mathcal{B}$ is ______________

2. Let A and B be 4×4 matrices with $\det(A) = 3$, $\det(B) = -4$. Find

(a) $\det(2A)$ (b) $\det(AB)$ (c) $\det(A^{-1})$

3. Let

$$A = \begin{pmatrix} 1 & -1 & 2 & 2 & 3 \\ 2 & -1 & 4 & 3 & 7 \\ 1 & -2 & 2 & 3 & 1 \\ 1 & 1 & 2 & 0 & 7 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -1 & 2 & 2 & 3 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

The matrices A and B are row equivalent. Find

(a) the rank of A.

(b) a basis for the column space of A.

(c) the dimension of the nullspace of A.

(d) a basis for the nullspace of A.

(e) a basis for the row space of A.

4. Let W be the subspace of $\mathbf{R}^3$ consisting of all vectors whose second and third components are equal. Find the dimension of W and a basis for W.

5. Let $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$ and $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}\}$ be bases for a vector space V, and suppose that $\mathbf{b_1} = 2\mathbf{c_1} + 3\mathbf{c_2}$ and $\mathbf{b_2} = 3\mathbf{c_1} + 5\mathbf{c_2}$.

(a) Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$.

(b) Find $[\mathbf{x}]_\mathcal{B}$ for $\mathbf{x} = 3\mathbf{c_1} + 4\mathbf{c_2}$.

6. A certain basketball player is a poor foul shooter. He finds that the outcome of a foul shoot depends on the result of the previous shot. (This is definitely not true for a good shooter.) To be precise, if he makes a foul shot, he will make the next one 60% of the time while if he misses a foul shot, he will miss the next one 70% of the time. If he takes a large number of foul shots, what percentage will he make?