MATH 240 Dr. Wolfe SAMPLE THIRD HOUR EXAM FALL 2008

1. Complete the following definition: The $n \times n$ matrix B is *diagonalizable* if ________

2. Define $T : \mathbf{R}^2 \to \mathbf{R}^2$ by $T(\mathbf{x}) = B\mathbf{x}$ where $B = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$. Find a basis $\mathcal{B}$ for $\mathbf{R}^2$ with the property that $[T]_{\mathcal{B}}$ is diagonal.

3. Let B be as in problem 2. Find the solution of the initial value problem $x' = Bx,\ x(0) = (1,4)^T$.

4. Let

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}, \quad V = \text{col } A.$$

(a) Apply the Gram-Schmidt process to find an orthonormal basis for V.
(b) Find a matrix C such that $V^{\perp}$ is the nullspace of C.
(c) Find $\dim V^{\perp}$ and a basis for $V^{\perp}$.
(d) Let $\mathbf{b} = (-1,5,7)^T$. Find $\text{Proj}_V \mathbf{b}$
(e) Show that $\mathbf{b} - \text{Proj}_V \mathbf{b}$ is in $V^{\perp}$.

5. Let A and $\mathbf{b}$ be as in Problem 4.
(a) Find the least squares solution of the equation $A\mathbf{x} = \mathbf{b}$.
(b) There is a simple relation between the solutions of problems 4d and part (a). Can you find it ?

6. Let $\{\mathbf{u_1}, \mathbf{u_2}, \mathbf{u_3}\}$ be an orthonormal set in $\mathbf{R^n}$ and let

$$\mathbf{u} = \mathbf{u_1} + 2\mathbf{u_2} + 2\mathbf{u_3} \text{ and } \mathbf{v} = \mathbf{u_1} + 7\mathbf{u_3}.$$

Determine $\mathbf{u} \cdot \mathbf{v}$, $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

7. Let A be a nonsingular matrix and let λ be an eigenvalue of A. Show that $1/\lambda$ is an eigenvalue of A^{-1}.