MATH 241H Dr. Wolfe SAMPLE SECOND HOUR EXAM Fall 2010

The Use of Calculators Is Not Permitted On This Exam

1. Let $f(x, y, z) = x^2 + y^2 - z$.

(a) Sketch and describe Σ, the level surface of f which passes through the point $P_0 = (1, 2, 6)$.

(b) Find an equation of the plane tangent to Σ at the point P_0.

(c) If $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find the directional derivative of f at P_0 in the direction of $\mathbf{a}$.

(d) Find the direction in which f increases most rapidly at P_0 and find the maximal directional derivative at that point.

(e) Let $\mathbf{b}$ be a vector which is tangent to Σ at P_0. What is the directional derivative of f at P_0 in the direction of $\mathbf{b}$? Explain.

2. Suppose z is a function of x and y and at the point $(x, y) = (2, 5)$ we have $\nabla z = 10\mathbf{i} - \mathbf{j}$. Suppose $x = u^2 v,\ y = 3u + uv$. What is $\partial z/\partial u$ at the point $(u, v) = (1, 2)$?

3.
By using an appropriate tangent plane for the function $g(x, y) = \sqrt{7 - x^2 - 2y^2}$ find an approximate value of $f(1.03, .98)$. The exact value is 2.004569779.

4. A wire 120 cm long is cut into three pieces of lengths x, y and $120 - x - y$, and each piece is bent into the shape of a square. Let $f(x, y)$ denote the sum of the areas of these squares. Show that the single critical point of f is a local minimum. But surely it is possible to *maximize* the sum of the areas. Explain.

5. The Maryland Widget Company has three factories, each manufacturing widgets. If the Baltimore factory produces x widgets, the Hagerstown factory produces y widgets and the Easton factory produces z widgets, their respective manufacturing costs are $(3x^2 + 200)$ dollars, $(y^2 + 400)$ dollars and $(2z^2 + 300)$ dollars. If an order of 1100 widgets is to be filled, use the method of Lagrange multipliers to determine how the production should be distributed among the three factories in order to minimize the total manufacturing cost.