MATH 241H Dr. Wolfe SAMPLE FOURTH HOUR EXAM Fall, 2010

The Use of Calculators Is Not Permitted On This Exam

1. Compute $\int_C e^x \cos z dx + y dy - e^x \sin z dz$ where C is parameterized by $\mathbf{r}(\mathrm{t}) = \cos^3 \pi \mathrm{t}\mathbf{i} + \sin^3 \pi \mathrm{t}\mathbf{j} + \pi \mathrm{t}^2\mathbf{k}$ for $0 \leq \mathrm{t} \leq 1/2$.

2. Compute $\int_C (y + e^x)\, dx + (2x^2 + \cos y)\, dy$ where C is the boundary of the triangle with vertices $(0, 0)$, $(1, 1)$ and $(2, 0)$ oriented counterclockwise.

3. Use Stokes's Theorem to compute $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = 2z\mathbf{i} + x\mathbf{j} + 3y\mathbf{k};$$

C is the ellipse in which the plane $z = x$ meets the cylinder $x^2 + y^2 = 4$, oriented counterclockwise as viewed from above.

4. Let D be the solid region containing those points whose distance from the origin is between 1 and 2. Let Σ be the boundary surface of D with outward normal $\mathbf{n}$. Let

$$\mathbf{F}(x, y, z) = (x^3 + 3xy^2)\mathbf{i} + (e^{xz} - x^2)\mathbf{j} + (z^3 + z^2)\mathbf{k}$$

Evaluate

$$\int \int_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS.$$