Dr. Wolfe MATH 241H MATLAB PROJECT #5 Due December 6, 2010

In this assignment we investigate vector fields and work integrals.

1. Consider the vector field

$$\mathbf{F}(x, y, z) = \mathbf{i} + (x + y^2)\mathbf{j} + z\mathbf{k}.$$

We shall display this vector field in the rectangle

$$R = \{-2 \le x \le 3,\ -1 \le y \le 2,\ -1 \le z \le 1\}.$$

Here is the script to do this.

```
u=@(x,y,z) 1+0*x;
v=@(x,y,z) x+y.^2;
w=@(x,y,z) z;
x=linspace(−2, 3, 6);
y=linspace(−1, 2, 6);
z=linspace(−1, 1, 6);
[X,Y,Z]=meshgrid(x,y,z);
U=u(X,Y,Z);
V=v(X,Y,Z);
W=w(X,Y,Z);
quiver3(X,Y,Z,U,V,W)
```

For use in the next few problems, we will need the mfile **simpvec**. So write the following script and save it as **simpvec.m**.

```
function s=simpvec(n)
s=2*ones(1,n+1);
s(2:2:n)=4*ones(1,n/2);
s(1)=1; s(n+1)=1;
```

2. We use Simpson's rule to estimate the value of the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$, where

$$\mathbf{F}(x, y, z) = x\mathbf{i} + z\mathbf{j} + e^{x+y}\mathbf{k}$$

and C is parameterized by $\mathbf{r}(t) = t\cos t\mathbf{i} + t\sin t\mathbf{j} + t^2\mathbf{k},\ 0 \le t \le 2\pi$. A call is made to the mfile **simpvec**.

```
% Define the components of F.
u=@(x,y,z) x;
v=@(x,y,z) z;
w=@(x,y,z) exp(x+y);
% Choose the t values for Simpson's rule.
n=200;
```

```
t=linspace(0,2*pi,n+1); dt=2*pi/n;
s=simpvec(n);
% Calculate the x,y,z values along the curve.
x=t.*cos(t); y=t.*sin(t); z=t.^2;
% Calculate the components of rdot along the curve
xdot=cos(t)−t.*sin(t);
ydot=sin(t)+t.*cos(t);
zdot=2*t;
% Calculate the terms of the intgrand
I1=u(x,y,z).*xdot;
I2=v(x,y,z).*ydot;
I3=w(x,y,z).*zdot;
% Compute the integal using simpson.
integral =dot(s,(I1+I2+I3))*dt/3
```

3.

(a) Let $\mathbf{F}(x, y) = xy\mathbf{i} + \cos(xy)\mathbf{j}$. Use **quiver** to graph the vector field on the square $0 \le x, y \le 4$.

(b) Let C be parametrized by $\mathbf{r}(t) = t^2\mathbf{i}+e^t\mathbf{j}$, $0 \le t \le 2$. Following the pattern of Exercise 2, make a numerical estimate of the line integral $\int_c \mathbf{F} \cdot \mathbf{dr}$. Use Simpson's rule, first with 100 subdivisions and then with 200 subdivisions. Estimate the error in the first calculation by using the fact that the second calculation should be far more accurate.

4.

(a) Let $\mathbf{F}(x, y, z) = (1 - y^2 - z^2)\mathbf{i}$. Use **quiver3** to graph the vector field on the cube $-1 \le x, y, z \le 1$.

(b) Let the curve C be parameterized by $\mathbf{r}(t) = t\mathbf{i}+t\cos t\mathbf{j}+t\sin t\mathbf{k}$, $0 \le t \le 2\pi$. Calculate the line integral $\int_C \mathbf{F} \cdot \mathbf{dr}$ by hand, and numerically with Simpson's rule.

mfile lint.m The mfile **lint.m** estimates the line integral of a two-dimensional vector field $\mathbf{F}(x, y) = u(x, y)\mathbf{i} + v(x, y)\mathbf{j}$ along a polygonal path determined by the user by clicking on the figure. The call is **lint(u,v,corners)**, where, as usual, **corners** is a vector $[a, b, c, d]$ that defines a rectangle R. You are then asked to enter the number N of line segments in the path. The file uses **quiver** to plot the vector field over R. Then click on the figure with the left mouse button to start the path of integration. A second click produces a line from the first point to the second point and computes the work done by the vector field along this line. This procedure can be repeated a total of N times. The cumulative value of the line integral is shown on the screen. The program also computes the line integral $\int_C x\,dy$. When the curve C is closed and forms a positively oriented polygon, this line integral is the area of the polygon.

. 5. Let

$$\mathbf{F}(x, y) = (x + \sin y)\mathbf{i} + x\cos y\mathbf{j}.$$

(a) Use the mfile **lint.m** with this $\mathbf{F}$ and the rectangle $R = [1, 4] \times [-2, 2]$. Calculate the work done by any two paths from the point $(1.5, -1)$ to the point $(3.5, 1)$. Use any number of segments. What are your results?

(b) Calculate the work around any closed path. What can you conclude about this vector field.