MATH 241H Dr. Wolfe SAMPLE FIRST HOUR EXAM Spring 2010

The Use of Calculators Is Not Permitted On This Exam

1.

(a) Find an equation of the plane containing the point $(2, 1, -1)$ which is parallel to the plane $2x + 3y - z = 4$.

(b) Find an equation of the line perpendicular to the planes in part (a) and passing through $(1, 2, 3)$. Give both a parametric form and a symmetric form.

2. Let $A = (1, 1, 0)$, $B = (0, 0, 1)$ and $C = (0, 1, 1)$. Find the area of the triangle whose vertices are A, B and C.

3. Find the distance D from the point $(1, -2, 5)$ to the plane

$$3(x - 1) - 4(y + 2) + 12z = 0.$$

4. The position vector of a particle is given by

$$\mathbf{r}(\mathrm{t}) = (\sin \mathrm{t} - \mathrm{t} \cos \mathrm{t})\mathbf{i} + (\cos \mathrm{t} + \mathrm{t} \sin \mathrm{t})\mathbf{j}.$$

(a) Find the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle.

Let C be the portion of the trajectory for which $2\pi \leq t \leq 4\pi$.

(b) Find the tangent vector $\mathbf{T}(\mathrm{t})$ and the normal vector $\mathbf{N}(\mathrm{t})$ for C.

(c) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, the tangential and normal components of the acceleration of the particle.

(d) Find the curvature of C.

(e) Find the length of C.

5. Find the position, velocity and speed of an object whose acceleration is $\mathbf{a} = 2\mathbf{i} + 6t\mathbf{j}$, initial position is $\mathbf{r}(0) = 2\mathbf{i}$ and whose initial velocity is $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$.