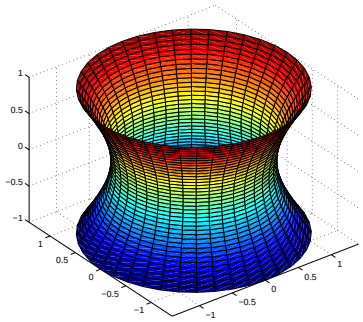
MATH 241H Dr. Wolfe SAMPLE SECOND HOUR EXAM Spring 2010

The Use of Calculators Is Not Permitted On This Exam

1. Multiple choice. The equation of the surface shown below is
(a) $x^2 + y^2 = z^2 - 1$ (b) $x^2 + y^2 = z^2$ (c) $x^2 + y^2 = z^2 + 1$



2. Let $w = f(x, y, z)$ be a differentiable function. Suppose that $f(8, -4, 1) = 5$ and $\nabla f(8, -4, 1) = \mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.
(a) Find the directional derivative of f at $(8, -4, 1)$ in the direction toward the origin.
(b) In what direction is the directional derivative of f at $(8, -4, 1)$ a maximum and what is the maximum value of the directional derivative ?
(c) Find an equation of the tangent plane to the level surface $f(x, y, z) = 5$ at $(8, -4, 1)$.
(d) If $x(t) = 8 + 2t, y(t) = 3t - 4, z(t) = 1 - \sin t$, what is $\frac{dw}{dt}$ at $t = 0$?

3. Let $z = \ln(x^2 + y^2)$. Show

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

4. Suppose $g(x, y)$ is a differentiable function such that $g(3, 4) = 5, \nabla g(3, 4) = 2\mathbf{i} - 5\mathbf{j}$. What is your best estimate for $g(2.98, 4.03)$?

5. (20 points) Let

$$f(x, y) = 3x^2 - 6xy + y^3 - 24y$$

Find <u>all</u> critical points of f. Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

6. (20 points) Use Lagrange multipliers to find the point (x, y, z) in the first octant $(x > 0, y > 0, z > 0)$ for which $x + y + z = 12$ and for which $f(x, y, z) = xy^2z^3$ is a maximum.