MATH 241H Dr. Wolfe SAMPLE FIRST HOUR EXAM Fall 2008

The Use of Calculators Is Not Permitted On This Exam

1. Let $\mathbf{A} = (4, 2, -1)$, $\mathbf{B} = (7, 1, 1)$, $\mathbf{C} = (12, 2, 1)$.

(a) Find parametric eqations for the line L containing $\mathbf{A}$ and $\mathbf{B}$.

(b) Find symmetric equations for the line through $\mathbf{C}$ parallel to L.

(c) Find an equation of the plane P containing $\mathbf{C}$ and perpendicular to L.

(d) Find $\mathbf{D}$, the point of intersection of the line L and the plane P.

(e) Find the distance from the point $\mathbf{A}$ to the plane P.

(f) Find the area A of the triangle whose vertices are $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.

2. The position vector of a particle is given by

$$\mathbf{R}(\mathrm{t}) = (\sin \mathrm{t} - \mathrm{t} \cos \mathrm{t})\mathbf{i} + (\cos \mathrm{t} + \mathrm{t} \sin \mathrm{t})\mathbf{j}.$$

Let C be the portion of the trajectory for which $2\pi \leq t \leq 4\pi$.

(a) Find the tangent vector $\mathbf{T}(\mathrm{t})$ and the normal vector $\mathbf{N}(\mathrm{t})$ for C.

(b) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, the tangential and normal components of the acceleration of the particle.

(c) Find the curvature of C.

(d) Find the length of C.

3. A ball rolls off a horizontal roof of a building 144 feet tall with a speed of 24 feet per second. How far away from the building is it when it hits the ground? Take $g = 32$ feet per second per second. (Note that the velocity vector is horizontal when the ball leaves the roof.)

4. Mark each statement as true (T) or false (F) (no reasons needed).

(i) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal unit vectors, $\mathbf{u} \times \mathbf{v}$ is a unit vector.

(ii) If $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors then $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w} = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$.

(iii) A vector-valued function $\mathbf{r}$ defined on an interval I is smooth if $\mathbf{r}$ has a continuous derivative on I.

(iv) If a smooth space curve C has its curvature $\kappa(t)$ identically zero then C is a line (or a line segment).

(v) If a particle moves with constant speed, its velocity and acceleration vectors are orthogonal.