MATH 241H Dr. Wolfe SAMPLE SECOND HOUR EXAM Fall 2008

The Use of Calculators Is Not Permitted On This Exam

1. Let $f(x, y, z) = x^2 + y^2 - z$.

(a) Sketch and describe $\Sigma$, the level surface of $f$ which passes through the point $P_0 = (2, 1, 6)$.

(b) Find an equation of the plane tangent to $\Sigma$ at the point $P_0$.

(c) If $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, find the directional derivative of $f$ at $P_0$ in the direction of $\mathbf{a}$.

(d) Find the direction in which $f$ increases most rapidly at $P_0$ and find the maximal directional derivative at that point.

(e) Let $\mathbf{b}$ be a vector which is tangent to $\Sigma$ at $P_0$. What is the directional derivative of $f$ at $P_0$ in the direction of $\mathbf{b}$? Explain.

(f) Let $\mathbf{r}(t) = g_1(t)\mathbf{i} + g_2(t)\mathbf{j} + g_3(t)\mathbf{k}$ be a smooth curve with

$$\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j} + 6\mathbf{k}, \qquad \frac{d\mathbf{r}}{dt}(1) = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}.$$

Let $u = f(x, y, z)$ and $x = g_1(t)$, $y = g_2(t)$, $z = g_3(t)$. Find $\frac{du}{dt}$ when $t = 1$.

2. Let $g(x, y) = xe^{xy}$. Show that $g_{xy} = g_{yx}$.

3. By using an appropriate tangent plane for the function $g(x, y) = \sqrt{x^2 + y}$, find an approximate value of $g(3.02, -4.98)$. The exact value is 2.034797287.

4. Let

$$f(x, y) = x^2 + 2xy + 3y^2 - 2x - 10y + 9.$$

Find all critical points of $f$. Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

5. The Ace Widget Company has determined that $x$ units of labor and $y$ units of capital can produce $f(x, y) = 60x^{3/4}y^{1/4}$ widgets. Also, suppose that each unit of labor costs \$100 while each unit of capital costs \$200. Assume that \$40,000 is available to spend on production. How many units of labor and how many units of capital should be utilized in order to maximize production ?