Dr. Wolfe MATH 241H MATLAB PROJECT #3 Due October 23, 2008

In this assignment we investigate functions of two variables. The purpose of the problems is to produce some nice pictures. Don't forget to label your graphs.

1. We wish to graph $f(x, y) = x^2 - y^2$ over the square $\{-2 \le x, y \le 2\}$. We do this as follows:

```
f=@(x,y) x.^2−y.^2;
% Set up a mesh for plotting.
x=−2:.1:2; y=−2:.1:2;
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
% The plotting command is surf.
surf(X,Y,Z)
```

2. Repeat problem 1 for
(a) $f(x, y) = \sin(x + y)$.
(b) $f(x, y) = \cos(x^2 + y^2)$.

3. Repeat problem 1 using the m-file **qsurf**. To see how to use it type help qsurf. Use a few different values of n.

4. We can also plot contour lines using the command **contour**. Let X, Y, Z be as in problem 1. Now do

```
level=−1.5:.3:1.5;
contour(X,Y,Z,level)
```

5. We find the tangent plane approximation to $f(x, y) = (1 - y^2)(1 - x^2)$ at the point $(x_0, y_0) = (.2, -.4)$. The partial derivatives are $f_x(x, y) = -2x(1 - y^2)$ and $f_y(x, y) = -2y(1 - x^2)$. Hence the tangent plane to the graph of f at $P_0 = (.2, -.4, f(.2, -.4))$ is

$$\begin{aligned} z = l(x, y) &= f(.2, -.4) + f_x(.2, -.4)(x - .2) + f_y(.2, -.4)(y + .4) \\ &= .8064 - .336(x - .2) + .768 * (y + .4), \end{aligned}$$

which has the nomal vector

$$\mathbf{N} = [-f_x(x_0, y_0), -f_y(x_0, y_0), 1] = [.336, -.768, 1].$$

Now we graph f over the square $\{-1 \le x, y \le 1\}$ and attach the tangent plane and normal vector. We graph the tangent plane over the smaller square $\{|x - .2|, |y + .4| \le .5\}$, and use a coarser mesh to make it more visible.

```
f= @(x,y) (1−x.^2).*(1−y.^2);
l=@(x,y) .8064−.336*(x−.2)+.768*(y+.4);
qsurf(f, [−1, 1, −1, 1])
hold on
```

```
qsurf(l,[-.3,.7,-.9,.1],10)
P=[.2,-.4,f(.2,-.4)]; N = [.336,-.768,1]; arrow3(P,N,'r')
hold off
```

6. We will now display a contour plot along with the gradient vector field. To display a vector field we use the command **quiver**.
Let $f(x, y) = xy - x^3/3$. Then $f_x(x, y) = y - x^2$ and $f_y = x$. We shall display the gradient vector field and the level curves of f over the square $[-2, 2] \times [-2, 2]$.

```
f=@(x,y) x.*y-(x.^3)/3;
fx=@(x,y) y-x.^2;
fy=@(x,y) x;
x=-2:.05:2;y=x;
% this is the fine mesh for the level curves.
[X,Y]=meshgrid(x,y);
Z=f(X,Y);
% We choose the level curves.
levels = [-6:.5:6];
contour(X,Y,Z,levels)
hold on
xx=-2:.2:2; yy=xx;
% This is the coarse mesh for the arrows
[XX,YY]=meshgrid(xx,yy);
U=fx(XX,YY); V=fy(XX,YY);
quiver(XX,YY,U,V)
axis equal
```

What is the relation between the level curves and the arrows?

7. Repeat problem 6 for $f(x, y) = x^2 + 4y^2$ Use the same square but you will need to consider a different set of level curves.