Dr. Wolfe MATH 241 MATLAB PROJECT #2 Due March 1, 2007

In this assignment we will inverstigate curves in two and three dimensions. For many of the problems, the output is a picture. Be sure to title the pictures using the **title** command.

1. We will first graph the curve $\mathbf{r}(t) = (\sin t - t\cos t)\mathbf{i} + (\cos t + t\sin t)\mathbf{j}$ for $0 \leq t \leq 4\pi$ together with some velocity vectors. So run the following script. (Note: a line starting with % is a comment and needn't be typed in.)

```
t=linspace(0,4*pi,121);
x=sin(t)-t.*cos(t);
y=cos(t)+t.*sin(t);
plot(x,y)
hold on
% Add the velocity vectors
for s=linspace(.16*pi,4*pi,24);
r=[sin(s)-s.*cos(s),cos(s)+s.*sin(s)];
u=[s.*sin(s),s.*cos(s)];
arrow(r,u,'r')
end
hold off
```

2. Repeat problem 1 for the parabola $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$ for $-2 \leq t \leq 2$.

3. Now we'll look at some space curves. We consider the helix $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t/(2*\pi)\mathbf{k}$ for $0 \leq t \leq 4\pi$. Run the following script.

```
t=linspace(0,4*pi,101);
x=inline('cos(t)');
y=inline('sin(t)');
z=inline('t/(2*pi)');
plot3(x(t),y(t),z(t))
hold on
% Add the velocity vectors.
for s=linspace(0,4*pi,17);
p=[x(s),y(s),z(s)];
v=[-sin(s),cos(s),1/(2*pi)];
arrow3(p,v,'r')
end
view(135,40)
hold off
```

4. Repeat problem 3 for the curve $r(t) = t\mathbf{i} + t^2/2\mathbf{j} + t^3/3\mathbf{k}$ for $-2 \leq t \leq 2$. If you follow the model of problem 3 remember to write "y=inline('t.^2/2')" etc.

5. We can also exhibit the Frenet frame. So execute the first 5 lines in problem 3. Then do

```
axis equal
hold on
frenet(x,y,z)
```

At this point you will get a prompt asking you to enter a value of t. Take $t = 0, .8\pi, 2.2\pi, 3.6\pi$. Then do

```
view(135,40)
```

to rotate the figure. The result should be very pretty.

Now we'll investigate arclength. Let the curve C be parameterized by

$$\mathbf{r}(t) = t\mathbf{i} + t^2/2\mathbf{j} + t^3/3\mathbf{k}, \qquad 0 \leq t \leq 2$$

6. First, we approximate the length of C by making a polygonal approximation.

```
t=0:.02:2;
x=t; y=t.^2/2; z=t.^3/3;
sum=0;
for j=1:100
    dx=x(j+1)-x(j);
    dy=y(j+1)-y(j);
    dz=z(j+1)-z(j);
    dr=[dx, dy, dz];
    sum=sum+norm(dr);
end
disp('Length of the polygonal appox. using 100 seqments')
sum
```

7. Next we will use the numerical integrator **quadl**. We have

$$\|\mathbf{r}'(t)\| = \sqrt{1 + t^2 + t^4}.$$

The arclength integral cannot be computed by hand. So do

```
speed=inline('sqrt(1+t.^2+t.^4)')
s=quadl(speed,0,2)
```

Compare your answer with the answer for problem 6.

8. Let the curve C be parameterized by

$$\mathbf{P}(t) = (1 - \cos t)\mathbf{i} + (1 + 2t + t^2)\mathbf{j}.$$

(a) Calculate by hand a tangent vector to the curve at $P(0)$.

(b) Use MATLAB to compute secant vectors $(\mathbf{P}(t) - \mathbf{P}(0))/t$ for $t = .2, .1, .05$. The error in the secant approximation is

$$\frac{\mathbf{P}(t) - \mathbf{P}(0)}{t} - \mathbf{P}'(0).$$

By what factor is the error in each component decreased when t is cut in half?

(c) Plot the curve C for $0 \le t \le 1$ and use the **arrow** feature to plot each of these secant vectors as well as the tangent vector computed in part (a). Attach each of these vectors to the point $\mathbf{P}(0) = (0, 1)$.