Dr. Wolfe MATH 241 MATLAB PROJECT #4 Due April 19, 2007

In this assignment we investigate functions of three variables. The graph of a function of three variables is a three dimensional surface in four dimensional space which we cannot visualize. One way to describe functions of three variables is in terms of their level surfaces. On the other hand, many surfaces in three-dimensional space are specified as level surfaces. We will also look at ways to display surfaces that are given parametrically.

mfile impl.m This mfile plots the level set $f(x, y, z) = c$. The ingredients for the use of this mfile are a function $f(x, y, z)$ given in an mfile or as an inline function, a set of coordinates defining a rectangular three-dimensional region, and a value c. The region is defined by a 6-vector "corners" which is equal to $[x_{min}, x_{max}, y_{min}, y_{max}, z_{min}, z_{max}]$.

We will use the command **subplot** which will enable us to put several graphs on the same figure and save on printing costs.

1. Consider $f(x, y, z) = x^2 + y^2 - z^2$, whose level sets are hyperboloids. We construct some level surfaces:

```
f=inline('x.^2+y.^2-z.^2','x','y','z')
corners=[-2 2 - 2 2 - 2 2];
subplot(2,2,1)
impl(f,corners,1)
title('c = 1')
subplot(2,2,2)
impl(f,corners,.1)
title('c = .1')
subplot(2,2,3)
impl(f,corners,0)
title('c = 0')
subplot(2,2,4)
impl(f,corners,-.5)
title('c = -.5')
```

2. Let $f(x, y, z) = x^2 + y^2 + z^2 + xy^2$. Use the mfile **impl** over the region $\{-4 \leq x, y \leq 1, -2 \leq z \leq 2\}$. Display the level sets $S_c = \{f(x, y, z) = c\}$ for $c = 1.2, 1, .8, .5, .3, .2$. Plot these graphs in the same figure window using the command **subplot** For each plot do **view(68,34)** to adjust the point-of-view. For what value of c does the level set S_c break into two components?

The gradient vector of a function of three variables is

$$\nabla f(x, y, z) = [f_x(x, y, z), f_y(x, y, z), f_z(x, y, z)].$$

The gradient vector at a point $P_0 = (x_0, y_0, z_0)$ is orthogonal to the level surface S through P_0. The gradient vector field can be displayed using the MATLAB command **quiver3**

3. Let $f(x, y, z) = z + (y^2 - x^2)/4$. The level surfaces of f are hyperbolic paraboloids. The gradient vector field is $\nabla f(x, y, z) = [-x/2, y/2, 1]$. Display the gradient vector field in two ways. First attach gradient vectors to points in a family of planes parallel to the xy plane at heights $z = -1, 0, 1$ over the square $0 \leq x, y \leq 2$.

```
[X,Y]= meshgrid(0:.4:2);
U=−X/2;
V=Y/2;
W=1+0*X;
subplot(1,2,1)
for z = [−1, 0, 1]
  Z=z+0*X;
 quiver3(X,Y,Z,U,V,W)
 hold on
end
axis image
```

Only three z levels were chosen; too many levels produce a very confusing picture. Next display the gradient vectors by attaching them to points on the portion of the level surface $f(x, y, z) = 0$ lying over the same square $0 \leq x, y \leq 2$. Thus add the following to the preceeding script

```
% First plot the surface with a finer mesh.
[XX,YY]=meshgrid(0: .05 : 2);
ZZ=.25*(XX.^2-YY.^2);
subplot(1,2,2)
surf(XX, YY, ZZ); shading interp
hold on
% Now add the vector field.
Z=.25*(X.^2-Y.^2);
quiver3(X,Y,Z,U,V,W)
axis image
```

Many surfaces are not graphs of functions but can be represented parametrically as

$$x = x(u, v), \qquad y = y(u, v) \qquad z = z(x, y)$$

where the points (u, v) lie in some region D of the plane. For instance the sphere of radius a can be represented as

$$x = a \sin u \cos v, \quad y = a \sin u \sin v, \quad z = a \cos u, \quad 0 \leq u \leq \pi, \quad 0 \leq v \leq 2\pi.$$

4. Graph the sphere of radius 2. Do

```
a=2;
u=linspace(0,pi,41);
v=linspace(0,2*pi,41);
[U,V]=meshgrid(u,v);
```

```
X=a*sin(U).*cos(V);
Y=a*sin(U).*sin(V);
Z=a*cos(U);
surf(X,Y,Z)
```

5. The torus is the surface of a doughnut. It is formed by rotating a circle about an axis which does not intersect the circle. The parametic equations of a torus are

$$x = \cos u(r + a\cos v), \qquad y = \sin u(r + a\cos v), \qquad z = a\sin v, \text{ with } r > a.$$

Draw the torus with $a = .5$, $r = 2$. Use the command **axis equal** to get the picture you want.

6. Now, just for fun, we'll draw a coffee (or beer) mug with a handle. We'll use the command **ezsurf**.

```
syms s t
% vertical cylinder of radius 1.
x=cos(s);
y=sin(s);
z=t;
ezsurf(x,y,z,[0 2*pi -2 2])
hold on
% handle formed by half of a torus with r=1, a=.25
% centered at (1,0,.5)
xhandle=1+cos(s)*(1+.25*cos(t));
yhandle=.25*sin(t)
zhandle= .5+sin(s)*(1+.25*cos(t));
ezsurf(xhandle,yhandle,zhandle,[-pi/2 pi/2 0 2*pi])
hold off
axis([-2 3 -2 2 -2 2])
view(34,34)
```