MATH 241 Dr. Wolfe SAMPLE FIRST HOUR EXAM SPRING 2006

The Use of Calculators Is Not Permitted On This Exam

1. Show that the points $(2,1,0), (0,2,-1)$ and $(6,-1,2)$ are colinear and find symmetric equations for the line containing them.
2. Find an equation of the plane containing the points $(1,0,-1), (-5,3,2)$ and $(2,-1,4)$.
3. Let $\mathbf{u} = 3\mathbf{i} + 5\mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$. Write $\mathbf{u} = \mathbf{u_1} + \mathbf{u_2}$ where $\mathbf{u_1}$ is parallel to $\mathbf{v}$ and $\mathbf{u_2}$ is perpendicular to $\mathbf{v}$.

4. Let $C$ be the curve.

$$\mathbf{r}(t) = (t^2+1)\mathbf{i} + (4t-3)\mathbf{j} + (2t^2-6t)\mathbf{k}.$$

   (a) Show that the point $P = (2,-7,8)$ lies on $C$.
   (b) Find the equation of the line tangent to $C$ at $P$. (Note: We didn't discuss tangent lines in class but it should be clear what this is.)
   (c) Write an integral which gives the length of that part of $C$ for which $1 \leq t \leq 2$. Do not evaluate the integral.
5. Find the position, velocity and speed of an object whose acceleration is $\mathbf{a} = -32\mathbf{k}$, initial position is $\mathbf{r_0} = 5\mathbf{j} + 2\mathbf{k}$ and whose initial velocity is $\mathbf{v_0} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

6. The trajectory of a particle is given by

$$\mathbf{r}(t) = 3\cos t\mathbf{i} + 3\sin t\mathbf{j} + 4t\mathbf{k}.$$

   (a) Find the tangent vector $\mathbf{T}(t)$ and the normal vector $\mathbf{N}(t)$ of the trajectory of the particle.
   (b) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, the tangential and normal components of the acceleration of the particle.
   (c) Find $\kappa(t)$, the curvature of the trajectory.