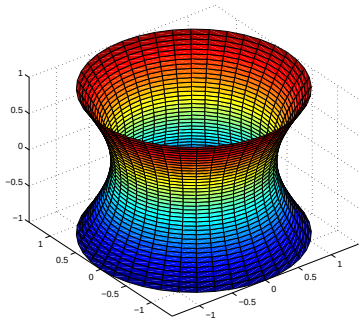
MATH 241 Dr. Wolfe SAMPLE SECOND HOUR EXAM SPRING 2006

The Use of Calculators Is Not Permitted On This Exam

1. Multiple choice. The equation of the graph shown below is
   (a) $x^2 + y^2 = z^2 - 1$ (b) $x^2 + y^2 = z^2$ (c) $x^2 + y^2 = z^2 + 1$



2. Let $w = f(x, y, z)$ be a differentiable function. Suppose that $f(4, 4, -2) = 6$ and $\nabla f(4, 4, -2) = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$.
   (a) Find the directional derivative of f at $(4, 4, -2)$ in the direction toward the origin.
   (b) In what direction is the directional derivative of f at $(4, 4, -2)$ a maximum and what is the maximum value of the directional derivative ?
   (c) Find an equation of the tangent plane to the level surface $f(x, y, z) = 6$ at $(4, 4, -2)$.
   (d) If $x(t) = 4 + 3t, y(t) = 4 - 2t, z(t) = -2 + t + t^2$, what is $\frac{dw}{dt}$ at $t = 0$?

3. Let $f(x, y) = \sin xy$. Compute all the second partial derivatives of f.

4. Suppose $g(x, y)$ is a differentiable function such that $g(2, 3) = 7, \nabla g(2, 3) = 2\mathbf{i} - 5\mathbf{j}$. What is your best estimate for $g(1.98, 3.03)$?

5. Let
$$f(x, y) = x^2y - 4xy + 2y^2 - 12y$$
   Find <u>all</u> critical points of f. Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

6. Suppose that a firm makes two products, widgets and flibbits, using the same raw materials. If x widgets and y flibbits are produced then x and y must satisfy the constraint $x^2 + 2y^2 = 8100$. (This expresses a limitation on the amount of raw materials available.) Each widget produces \$5 profit and each flibbit produces \$20 profit. How many of each product should the firm produce in order to maximize the profit ?