MATH 241 Dr. Wolfe SAMPLE FOURTH HOUR EXAM SPRING 2006

The Use of Calculators Is Not Permitted On This Exam

1. Let $\mathbf{F} = (2xz + y^2)\mathbf{i} + (z^2 + 2xy)\mathbf{j} + (2yz + x^2 + 1)\mathbf{k}$.
   (a) Show that F is conservative and find a function f such that $\mathbf{F} = \nabla f$.
   (b) Compute $\int_C \mathbf{F} \cdot \mathbf{dr}$ where C is the curve

$$x = t^5, \qquad y = 1 - \cos^3 \pi t/2, \qquad z = \sin^3 \pi t/2, \qquad 0 \le t \le 1.$$

2. Use Green's Theorem to compute

$$\int_C xydx + x^2dy$$

   where C is the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$. C is oriented counterclockwise.

3. Evaluate $\int\int_\Sigma \nabla \times \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = xz^2\mathbf{i} + x\mathbf{j} + \cos xz\mathbf{k}$ and Σ is the part of the ellipsoid $x^2 + y^2 + 3z^2 = 1$ above the xy plane. $\mathbf{n}$ is directed upward.

4. Evaluate $\int\int_\Sigma \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$ and Σ is the boundary of the solid region bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the plane $z = 1$. $\mathbf{n}$ points outward.