MATH 241 Dr. Wolfe SAMPLE SECOND HOUR EXAM SPRING 2007

The Use of Calculators Is Not Permitted On This Exam

1. Let $f(x, y, z) = x^2 + y^2 - z$.

(a) Sketch and describe Σ, the level surface of f which passes through the point $P_0 = (1, 2, 6)$.

(b) Find an equation of the plane tangent to Σ at the point P_0.

(c) If $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, find the directional derivative of f at P_0 in the direction of $\mathbf{a}$.

(d) Find the direction in which f increases most rapidly at P_0 and find the maximal directional derivative at that point.

(e) Let $\mathbf{b}$ be a vector which is tangent to Σ at P_0. What is the directional derivative of f at P_0 in the direction of $\mathbf{b}$? Explain.

2. Let

$$w = \ln(x^2 + y^2 + z^2), \quad x = \cos t, \quad y = \sin t, \quad z = t.$$

Find dw/dt in two ways. Check that they both give the same answer.

(a) Make the substitutions for x, y and z before differentiating.

(b) Use the chain rule.

3. By using an appropriate tangent plane for the function $g(x, y) = \sqrt{x^2 + y}$ find an approximate value of $g(3.02, -4.98)$. The exact value is 2.034797287.

4. Let

$$f(x, y) = x^2 + 4xy + 5y^2 - 10x - 22y + 11$$

Find all critical points of f. Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

5. The Ace Widget Company has determined that x units of labor and y units of capital can produce $f(x, y) = 60x^{3/4}y^{1/4}$ widgets. Also suppose that each unit of labor costs \$100 while each unit of capital costs \$200. Assume that \$40,000 is available to spend on production. How many units of labor and how many units of capital should be utilized in order to maximize production ?