MATH 241 Dr. Wolfe SAMPLE FOURTH HOUR EXAM SPRING 2007

The Use of Calculators Is Not Permitted On This Exam

1. Let $\mathbf{F} = y\sin z\mathbf{i} + (x\sin z + z)\mathbf{j} + (xy\cos z + y + 2z)\mathbf{k}$.
   (a) Show that F is conservative and find a function f such that $\mathbf{F} = \nabla f$.
   (b) Compute $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the curve

$$x = \frac{2t}{t^2+1}, \qquad y = t^{7/4}, \qquad z = \frac{\pi}{t^2+1}, \qquad 0 \le t \le 1.$$

   (The curve is oriented in the direction of increasing t.)

2. Use Green's Theorem to compute

$$\int_C x^2y\,dx + x^3\,dy$$

   where C is the boundary of the region in the first quadrant between the graphs of $y = x^2$ and $y = x$ oriented counterclockwise.

3. Use Stokes' theorem to evaluate

$$\int\int_\Sigma (\nabla \times \mathbf{F})\cdot \mathbf{n}\,dS$$

   where $\mathbf{F} = 3z\mathbf{i} + 5x\mathbf{j} - 2y\mathbf{k}$, Σ is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 4$ and $\mathbf{n}$ is the upward pointing unit normal to Σ.

4. Let Σ be the boundary of the region D in the first octant bounded above by the plane $z = 1$, below by the plane $z = 0$ and on the sides by the planes $x = 0, y = 0$ and $x + y = 1$. Let

$$\mathbf{G}(x,y,z) = x^2\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}.$$

   Compute

$$\int\int_\Sigma \mathbf{G}\cdot\mathbf{n}\,dS$$

   where $\mathbf{n}$ is the outward unit normal to Σ.