

The Use of Calculators Is Not Permitted On This Exam

1. Let $A = (3, 2, 1)$, $B = (1, 3, 0)$, $C = (7, 0, 3)$, $D = (0, 0, 1)$.

- (a) Show that the points A , B , and C are colinear and find symmetric equations for the line containing them.
- (b) Find an equation of the plane containing the points A , B and D .

2. Let $\mathbf{u} = 3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$, $\mathbf{v} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. Write $\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2$ where \mathbf{u}_1 is parallel to \mathbf{v} and \mathbf{u}_2 is perpendicular to \mathbf{v} .

3. Given the curve C :

$$\mathbf{r}(t) = (t^2 + 2)\mathbf{i} + (3t - 3)\mathbf{j} + (t^2 - 6t)\mathbf{k}.$$

- (a) Show that the point $P = (3, -6, 7)$ lies on C .
- (b) Find the equation of the line tangent to C at P . (Note: We didn't discuss tangent lines in class but it should be clear what this is.)
- (c) Write an integral which gives the length L of that part of C for which $1 \leq t \leq 2$. Do not evaluate the integral.

4. Find the position, velocity and speed of an object whose acceleration is $\mathbf{a} = -32\mathbf{k}$, initial position is $\mathbf{r}_0 = 4\mathbf{j} + 2\mathbf{k}$ and whose initial velocity is $\mathbf{v}_0 = 4\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

5. The trajectory of a particle is given by

$$\mathbf{r}(t) = 3 \cos t \mathbf{i} + 4t \mathbf{j} + 3 \sin t \mathbf{k}.$$

- (a) Find the tangent vector $\mathbf{T}(t)$ and the normal vector $\mathbf{N}(t)$ of the trajectory of the particle.
- (b) Find $a_{\mathbf{T}}$ and $a_{\mathbf{N}}$, the tangential and normal components of the acceleration of the particle.
- (c) Find $\kappa(t)$, the curvature of the trajectory.