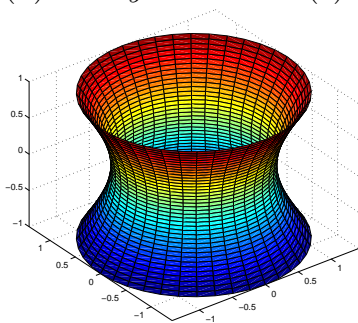


The Use of Calculators Is Not Permitted On This Exam

1. Multiple choice. The equation of the surface shown below is

- (a) $x^2 + y^2 = z^2 - 1$ (b) $x^2 + y^2 = z^2$ (c) $x^2 + y^2 = z^2 + 1$



2. Let $w = f(x, y, z)$ be a differentiable function. Suppose that $f(8, 4, -1) = 3$ and $\nabla f(8, 4, -1) = \mathbf{i} - 4\mathbf{j} + \mathbf{k}$.

- (a) Find the directional derivative of f at $(8, 4, -1)$ in the direction toward the origin.
 (b) In what direction is the directional derivative of f at $(8, 4, -1)$ a maximum and what is the maximum value of the directional derivative?
 (c) Find an equation of the tangent plane to the level surface $f(x, y, z) = 3$ at $(8, 4, -1)$.
 (d) If $x(t) = 8 + 2t$, $y(t) = 4 - 3t$, $z(t) = \sin t - 1$, what is $\frac{dw}{dt}$ at $t = 0$?

3. Let $f(x, y) = \sin xy^2$. Compute all the second partial derivatives of f .

4. Suppose $g(x, y)$ is a differentiable function such that $g(3, 4) = 5$, $\nabla g(3, 4) = 2\mathbf{i} - 5\mathbf{j}$. What is your best estimate for $g(2.98, 4.03)$?

5. Let

$$f(x, y) = x^2y + x^2 + y^2 - 2y + 1$$

Find all critical points of f . Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

6. The Maryland Widget Company has three factories, each manufacturing widgets. If the Baltimore factory produces x widgets, the Hagerstown factory produces y widgets and the Easton factory produces z widgets, their respective manufacturing costs are $(3x^2 + 200)$ dollars, $(y^2 + 400)$ dollars and $(2z^2 + 300)$ dollars. If an order of 1100 widgets is to be filled, use the method of Lagrange multipliers to determine how the production should be distributed among the three factories in order to minimize the total manufacturing cost.