

The Use of Calculators Is Not Permitted On This Exam

1. Consider the integral

$$I = \int_0^4 \int_{\sqrt{x}}^2 \sqrt{1+y^3} dy dx.$$

- (a) Sketch the region  $R$  over which the integration takes place.
- (b) Evaluate  $I$  by reversing the order of integration.

2. Set up a triple integral for finding the volume  $V$  of the solid bounded on the top by the plane  $z = y$ , on the bottom by the  $xy$  plane and on the sides by the plane  $y = 3x - 2$  and the parabolic sheet  $y = x^2$ . Do not evaluate the integral.

3. An object occupies the region bounded above by the sphere  $x^2 + y^2 + z^2 = 1$  and below by the cone  $z = \sqrt{x^2 + y^2}$  and has mass density

$$\delta(x, y, z) = z\sqrt{x^2 + y^2 + z^2}.$$

- (a) Find the mass of the object.
- (b) Find the center of gravity of the object.

4. Find the surface area  $S$  of the portion of the surface  $z = xy$  that is inside the cylinder  $x^2 + y^2 = 1$ .

5. Compute  $\int \int_R y dA$  where  $R$  is the region bounded by  $y = 3x$ ,  $x = 3y$  and  $x + y = 4$  by making the change of variables  $x = 3u + v$ ,  $y = u + 3v$ .