MATH 241H Dr. Wolfe SAMPLE FIRST HOUR EXAM Spring 2009

The Use of Calculators Is Not Permitted On This Exam

1. Let $A = (0, 0, 0)$, $B = (1, 1, 0)$, $C = (2, 0, 1)$, $D = (1, -1, 1)$.

(a) Show that these four points lie on a plane $\mathcal{P}$ and find an equation of $\mathcal{P}$.

(b) Let $\mathcal{Q}$ be the quadralateral whose vertices are A, B, C and D (in that order). Show that $\mathcal{Q}$ is a parallelogram.

(c) Show that $\mathcal{Q}$ is a rectangle.

(d) Is $\mathcal{Q}$ a square? Explain.

(e) Find the area of $\mathcal{Q}$.

(f) Let $E = (1, 2, 3)$. Find the distance from E to $\mathcal{P}$.

2. Find the position, velocity and speed of an object whose acceleration is

$$\mathbf{a} = -\cos t\,\mathbf{i} - \sin t\,\mathbf{j},$$

initial position is $\mathbf{r_0} = \mathbf{i}$ and whose initial velocity is $\mathbf{v_0} = \mathbf{k}$.

3. The position of a moving particle is given by

$$\mathbf{r}(t) = \frac{t^3}{3}\,\mathbf{i} + t^2\,\mathbf{j} + 2t\,\mathbf{k} \text{ for } 0 \leq t \leq 2.$$

(a) Find the velocity, speed, and the tangential and normal components of the acceleration of the particle for any time t with $0 \leq t \leq 2$.

(b) Find the curvature κ of the trajectory of the particle at any time t with $0 \leq t \leq 2$.

(c) Find the total distance travelled by the particle in the given time interval.