

## The Use of Calculators Is Not Permitted On This Exam

1. Let  $f(x, y, z) = x^2 + y^2 - z$ .
  - (a) Sketch and describe  $\Sigma$ , the level surface of  $f$  which passes through the point  $P_0 = (2, 1, 6)$ .
  - (b) Find an equation of the plane tangent to  $\Sigma$  at the point  $P_0$ .
  - (c) If  $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ , find the directional derivative of  $f$  at  $P_0$  in the direction of  $\mathbf{a}$ .
  - (d) Find the direction in which  $f$  increases most rapidly at  $P_0$  and find the maximal directional derivative at that point.
  - (e) Let  $\mathbf{b}$  be a vector which is tangent to  $\Sigma$  at  $P_0$ . What is the directional derivative of  $f$  at  $P_0$  in the direction of  $\mathbf{b}$ ? Explain.
  - (f) Let  $\mathbf{r}(t) = g_1(t)\mathbf{i} + g_2(t)\mathbf{j} + g_3(t)\mathbf{k}$  be a smooth curve with

$$\mathbf{r}(1) = 2\mathbf{i} + \mathbf{j} + 6\mathbf{k}, \quad \frac{d\mathbf{r}}{dt}(1) = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}.$$

Let  $u = f(x, y, z)$  and  $x = g_1(t)$ ,  $y = g_2(t)$ ,  $z = g_3(t)$ . Find  $\frac{du}{dt}$  when  $t = 1$ .

2. Let  $g(x, y) = xe^{xy}$ . Show that  $g_{xy} = g_{yx}$ .
3. By using an appropriate tangent plane for the function  $g(x, y) = \sqrt{x^2 + y}$ , find an approximate value of  $g(3.02, -4.98)$ . The exact value is 2.034797287.

4. Let

$$f(x, y) = x^2 + 2xy + 3y^2 - 2x - 10y + 9.$$

Find all critical points of  $f$ . Determine whether each critical point yields a relative maximum, a relative minimum or a saddle point.

5. The Ace Widget Company has determined that  $x$  units of labor and  $y$  units of capital can produce  $f(x, y) = 60x^{3/4}y^{1/4}$  widgets. Also, suppose that each unit of labor costs \$100 while each unit of capital costs \$200. Assume that \$40,000 is available to spend on production. How many units of labor and how many units of capital should be utilized in order to maximize production ?