MATH 241H Dr. Wolfe SAMPLE FOURTH HOUR EXAM Spring 2009

The Use of Calculators Is Not Permitted On This Exam

1. Let $\mathbf{F} = y \sin z\mathbf{i} + (x \sin z + z)\mathbf{j} + (xy \cos z + y + 2z)\mathbf{k}$.
(a) Show that F is conservative and find a function f such that $\mathbf{F} = \nabla f$.
(b) Compute $\int_C \mathbf{F} \cdot \mathbf{dr}$ where C is the curve

$$x = \frac{2t}{t^2+1}, \qquad y = t^{7/4}, \qquad z = \frac{\pi}{t^2+1}, \qquad 0 \le t \le 1.$$

(The curve is oriented in the direction of increasing t.)

2. Compute

$$\int_C 2y^3\, dx + (x^4 + 6y^2 x)\, dy$$

where C is the boundary of the region in the first quadrant bounded by $y = 0$, $x = 0$ and $x^4 + y^4 = 1$, oriented counterclockwise.

3. Use Stokes' theorem to evaluate

$$\int\int_\Sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = 3z\mathbf{i} + 5x\mathbf{j} - 2y\mathbf{k}$, Σ is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 4$ and $\mathbf{n}$ is the upward pointing unit normal to Σ.

4. Let Σ be the boundary of the region D in the first octant bounded above by the plane $z = 1$, below by the plane $z = 0$ and on the sides by the planes $x = 0, y = 0$ and $x + y = 1$. Let

$$\mathbf{G}(x, y, z) = x^2\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}.$$

Compute

$$\int\int_\Sigma \mathbf{G} \cdot \mathbf{n}\, dS$$

where $\mathbf{n}$ is the outward unit normal to Σ.