MATH 401 Dr. Wolfe ASSIGNMENT #2 Due October 20, 2003

1. P3.2.14, p.152 **Strang.**

2. P3.3.24, p.165 **Strang.**

3. P3.4.6, p.180 **Strang.**

4. The *Legendre Polynomials*, $P_n(x)$ are the set of polynomials orthogonal over $[-1,1]$ normalized so that $P_n(1) = 1$. The first two Legendre Polynomials are $P_0(x) = 1$, $P_1(x) = x$.
   (a) Find $P_2(x)$.
   (b) Find the polynomial $q_2(x)$ of degree ≤ 2 which is the best approximation to $f(x) = e^x$ in the sense of least squares on the interval $[-1,1]$.
   (c) Plot e^x and $q_2(x)$ together on $[-1,1]$
   Note: You can use MATLAB to compute the integrals if you like. MATLAB can do symbolic integration.

5. P3.4.27, p.182 **Strang.**

6. (MATLAB) In a paper dealing with the efficiency of energy utilization of the larvae of the Modest Sphinx moth (*Pachysphinx modesta*), L. Schroeder used the following data to determine a quadratic least-squares log-log relation between W, the live weight of the larvae in grams, and R, the oxygen consumption of the larvae in ml/hr. The form of the relation was

$$\log R = a + b \log W + c(\log W)^2,$$

   where a, b and c are constants determined by the data. Here $\log W$ is the common logarithm (MATLAB: log10).

W	R	W	R	W	R
0.017	0.154	0.783	1.47	0.111	0.357
0.233	0.537	2.75	1.84	1.11	0.531
1.32	1.15	3.02	2.01	1.69	1.44
4.29	3.40	5.45	3.52	4.83	4.66

   Form the 12×3 data matrix A and find the vector $(a, b, c)^T$ in 4 different ways.
   (a) By using the backslash operator.
   (b) By forming and solving the normal equations. Note the condition number of the matrix A^TA.
   (c) By using the QR decomposition.
   (d) By using the Singular-Value Decomposition. All this is quite easy in MATLAB. Plot the graph of R versus W with the values of a, b and c you found. On the same graph plot the data points. Note: if the data is represented as vectors R and W, to plot them do "plot(R, W, 'o')".

7. Let

$$f(x) = \begin{cases} -1 & -\pi \leq x \leq 0, \\ 1 & 0 < x \leq \pi. \end{cases}$$

(a) Expand f in a Fourier Series. Since f is an odd function there will only by terms involving $\sin nx$.
(b) Plot $f(x)$ and the first few partial sums on the same graph. Note: in MATLAB, once you have defined a vector of x-values (for example x=linspace(−pi,pi,101)), an easy way to create the function values is $f(x)$ is " $f(x) = (x > 0) - (x < 0)$".
(c) How many term are need in the sum to get an error $< .1$?

8. Let

$$f(t) = 2\exp(-2it) - 1 + \exp(3it).$$

(a) Let $N = 4$. Write out the equations for the coefficients $d_k, k = 0, 1, 2, 3$. Solve for d_k by inverting the Fourier matrix F_4.
(b) Write out the real and imaginary parts of the DFT approximation

$$g(t) = d_0 + d_3 e^{-it} + d_1 e^{it} + d_2 e^{2it}.$$

Use MATLAB to plot the real parts of f and g together on $[0, 2\pi]$ in one graph, and the imaginary parts of f and g in another. Circle the points where g interpolates f.
(c) Now let $N = 6$. Find the primative complex root of unity W_6. Use MATLAB to calculate the Fourier matrix F_6.
(d) Make an inline function for f. Compute g, the DFT approximation for f. by using the following instructions.

```
  sample = linspace(0, 2*pi, 7)
% We do not use the last sample point which is 2*pi
% transpose makes it into a column vector
   tt=transpose(sample(1:6));
% ff is also a column vector
ff=f(tt);
% Enter the Fourier matrix F = F_6 here.
dd=F\ ff;
d0=dd(1);
d=dd(2:N);
Sum up the 6 terms in g using the coefficients d(k).
```

Compare $g(t)$ and $f(t)$. They should agree.

9. Let $f(t) = \exp(\sin(2t)) + \cos(10t)$. Download the file dft.m from the class webpage.
(a) Use the program first with $N = 6$. Print out the graphs. Circle the points where g interpolates f.
(b) Keep increasing N until you get a good fit for both the real and imaginary parts. How large does N have to be ? How do the number of sampling points relate to the number of oscillations of f in the interval $[0, 2\pi]$?

10 Take the function of Problem 7 extended periodically. Apply dft with various values of N until you get a good fit. How large does N have to be ?