MATH 401 SAMPLE EXAM

1. Let A be a 3×3 matrix whose eigenvalues are $1, 2$ and -4.
   (a) Show that A is nonsingular.
   (b) Is A necessarily diagonalizable ? Explain.
   (c) What are the eigenvalues of A^{-1} ?
   (d) What are the eigenvalues of A^2 ?

2. Let
$$A = \begin{pmatrix} 10 & -6 \\ -6 & 10 \end{pmatrix}$$
   Find a symmetric, positive definite matrix B such that $B^2 = A$.

3. Let $f(x, y) = x^2 + 6xy + 2y^2 - 6x + 10y - 2$.
   (a) Find the critical point(s) for f and determine whether each critical point yields a relative minimum, a relative maximum or a saddle point for f.
   (b) Complete the following sentence: The graph of $f(x, y) = 0$ is a (an) _____.

4.
   (a) Find the general solution in terms of <u>real</u> functions of the system
$$x' = x - 5y,$$
$$y' = x - 3y$$
   (b) Find the solution of the system in part (a) satisfying
$$x(0) = 1, \qquad y(0) = 1.$$

5. A certain basketball player is a poor shooter. He finds that the outcome of a shot depends on the result of the previous shot. (This is definitely not true for a good shooter.) To be precise, if he makes a shot he will make the next shot 60% of the time while if he misses a shot he will miss the next one 70% of the time. If he takes a large number of shots what percentage will he make ?