

1. Problem 24, sec. 6.4, **Lay**. Find an orthonormal basis for the column space of A . Since you are using MATLAB, you can normalize the basis vectors as soon as you find them.
2. Problem 25, sec. 6.4, **Lay**. Note: If you have A and Q , R can be found with exactly four keystrokes.
3. Problem 13, sec. 6.6, **Lay**. Form the 13×4 data matrix A and find the vector $(\beta_0, \beta_1, \beta_2, \beta_3)^T$ in 4 different ways.
 - (a) By using the backslash operator.
 - (b) By forming and solving the normal equations.
 - (c) By using the QR decomposition.
 - (d) By using the Singular-Value Decomposition. All this is quite easy in MATLAB. Plot the graph of y versus t with the values of $\beta_0, \beta_1, \beta_2$ and β_3 you found. On the same graph plot the data points. Note: If the data is represented as vectors t and y , to plot them do “plot(t, y, 'o')”.
4. Problem 26, sec. 7.4, **Lay**. Use the **eig** command. Display your result in the usual 'format short'.
4. Use the program `pix.m` which you can find on the class webpage. You have three data choices. You must also enter the number of subdivisions in the mesh. Then you can approximate the original image by adding up to N terms of the singular value expansion of the image matrix.
 - (a) Enter 10 for the mesh size. Then choose data choice 1. You will see a random 10×10 pattern of gray, white and black squares. This is the original picture. Now you can approximate this picture by adding up the terms of the SVD expansion of the matrix. How many terms in the expansion are necessary to get a recognizable approximation? Note how fast the singular values are decreasing.
 - (b) Enter 40 for the mesh size. Then choose data choice 2. Now how many terms are necessary to get a good approximation of the original image? Compare the rate of decrease of the singular values of this image with the rate of decrease in part a). If you use only 4 terms in the expansion, how much do you save in storage space as compared to the storage space for the full 40×40 matrix?
 - (c) Enter 100 for the mesh size. Choose data choice 3. This is as close as I can come to a fingerprint. Again note how many terms are necessary to get a good approximation. Relate this observation to the rate at which the singular values decrease. Compare with the rates of parts a) and b). How much storage space do you save if you use 20 terms in the expansion as compared to the full 100×100 matrix ?
 - (d) Make up your own image matrix Z and insert it in the line for data choice 4. Analyse it as in question (a).

5. (MATLAB) Let

$$A = \begin{bmatrix} 8 & -4 & 4 \\ -4 & 6 & 0 \\ 4 & 0 & 10 \end{bmatrix}.$$

- (a) Use the power method to find the dominant eigenvalue of A .
- (b) Use the unshifted QR algorithm: $A_0 = A, A_k = Q_k R_k, A_{k+1} = R_k Q_k$ to find the eigenvalues of A .
- (c) Use the explicitly shifted QR algorithm: $A_0 = A, A_k - A_k(3,3)I = Q_k R_k, A_{k+1} = R_k Q_k + A_k(3,3)I$ to find the eigenvalues of A . When the third row and column of A_k are essentially zero except for the (3,3) entry 'deflate' the problem by working with the 2×2 principal submatrix.