MATH 401 Dr. Wolfe ASSIGNMENT #1 Due February 14, 2011

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 5 & -2 & 0 \\ 2 & 4 & -2 & 1 \\ 4 & 2 & -5 & 1 \end{bmatrix}.$$

   (a) Use the MATLAB command $[L, U, P] = \text{lu}(A)$ to get the factorization $PA = LU$. Verify that indeed $P * A = L * U$.
   (b) Now let $B = PA$. Try the command $[L, U, P] = \text{lu}(B)$. Why is $P = I$?

2. Ex. 15, p.41 **Strang.**

3. Ex. 42, p.55 **Strang.**

4. Recall that a square matrix A is <u>symmetric</u> if $A = A^T$. Prove that if A is $m \times n$, AA^T and A^TA are always symmetric. Show by example that they may not be equal, even for square matrices.

5. Let

$$A = \begin{pmatrix} 4 & -2 & -4 & 2 \\ -2 & 10 & -7 & 5 \\ -4 & -7 & 17 & -2 \\ 2 & 5 & -2 & 30 \end{pmatrix}$$

   (a) Find (by hand) a lower triangular matrix L such that $A = LL^T$ (Cholesky factorization) You may use the MATLAB command **chol** to check your work.
   (b) Let $\mathbf{b} = (-6, -45, 54, -67)^T$. Use the result of (a) to solve $A\mathbf{x} = \mathbf{b}$ by forward elimination and back substitution.

6. The solution of the boundary value problem

$$-\frac{d^2u}{dx^2} = 4\pi^2 \sin 2\pi x, \qquad u(0) = 0, \quad u(1) = 0$$

   is $u(x) = \sin 2\pi x$. We wish to construct finite difference approximations to $u(x)$ as on p. 59 of Strang. We will use MATLAB for this.
   (a) Download the M-file trid.m from the class website. This is a fast tridiagonal solver. Use it to find solutions to the system of finite difference equations for $n = 9, 19, 39$.
   (b) Evaluate the accuracy of your solutions by taking the norm of the difference of the solution vector and the vector of exact solutions values at the nodes.

Start your MATLAB session by typing **format long e**. This formats the output properly. Although you may discuss the problems with your classmates, all work should be your own. Cooperative efforts are not permitted.