MATH 401 Dr. Wolfe ASSIGNMENT #3 Due April 6, 2011

1. Ex. 18, p.242 **Strang.**
2. Ex 4, p.263 **Strang.**
3. Ex. 8, p.276 **Strang.**
4. Ex. 15 p.308 **Strang.** (Do it by hand. You may use MATLAB as a check.)
5. (MATLAB) "All Gaul is divided into three parts." Call them East Gaul, Central Gaul and West Gaul. Each year 5% of the residents of East Gaul move to Certral Gaul and 5% move to West Gaul. Of the residents of Central Gaul 15% move to East Gaul and 10% move to West Gaul. And of the residents of West Gaul, 10% move to East Gaul and 5% move to Central Gaul. What percentage of the population resides in each of the three regions after a long period of time ?
6. (MATLAB) Solve $\mathbf{x}' = A\mathbf{x},\ \mathbf{x}(0) = \mathbf{x_0}$ where
$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}, \quad \mathbf{x_0} = \begin{bmatrix} 6 \\ -4 \\ 4 \end{bmatrix}.$$
7. Find the general solution of $\mathbf{x}' = B\mathbf{x}$ where
$$B = \begin{pmatrix} 0 & 4 \\ -1 & -4 \end{pmatrix}$$
8. Solve $\mathbf{x}' = C\mathbf{x},\ \mathbf{x}(0) = \mathbf{x_0}$ where
$$C = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{x_0} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$
9. A matrix A is <u>nilpotent</u> if $A^k = 0$ for some positve integer k. Show that if A is nilpotent its only eigenvalue is zero. (Note: The converse of this is also true.)
10.
    (a) Suppose A is a diagonalizable matrix, all of whose eigenvalues have absolute value less than 1. Show that $A^n \to 0$ as $n \to \infty$.
    (b) Let A be as in part (a). Show that $I - A$ is invertible and
    $$(I - A)^{-1} = I + A + A^2 + A^3 + \cdots.$$
    (c) Let
    $$A = \begin{pmatrix} .2 & -.2 \\ .3 & .7 \end{pmatrix}.$$
    Use MATLAB to sum the series and verify the result of part (b). For this set $I = \text{eye}(2), S = I, S = I + S * A$. Now by using the "up-arrow" repeat the last command until you have convergence. Compare with $(I - A)^{-1}$.