MATH 401 Dr. Wolfe ASSIGNMENT #1 Due June 9, 2004

1. Let A be the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 4 & 5 & -2 & 0 \\ 2 & 4 & -2 & 1 \\ 4 & 2 & -5 & 1 \end{bmatrix}.$$

   (a) Use the MATLAB command $[L, U, P] = \mathrm{lu}(A)$ to get the factorization $PA = LU$. Verify that indeed $P * A = L * U$.
   (b) Now let $B = PA$. Try the command $[L, U, P] = \mathrm{lu}(B)$. Why is $P = I$?

2. P1.5.12, p.41 **Strang.**

3. P1.5.15, p.41 **Strang.**

4. P1.6.14, p.50 **Strang.**

5. Let

$$A = \begin{pmatrix} 4 & -2 & -4 & 2 \\ -2 & 10 & -7 & 5 \\ -4 & -7 & 17 & -2 \\ 2 & 5 & -2 & 30 \end{pmatrix}$$

   (a) Find (by hand) a lower triangular matrix L such that $A = LL^T$ (Cholesky factorization) You may use the MATLAB command **chol** to check your work.
   (b) Let $\mathbf{b} = (-6, -45, 54, -67)^T$. Use the result of (a) to solve $A\mathbf{x} = \mathbf{b}$ by forward elimination and back substitution.

6. The solution of the boundary value problem

$$-\frac{d^2u}{dx^2} = 4\pi^2 \sin 2\pi x, \qquad u(0) = 0, \quad u(1) = 0$$

   is $u(x) = \sin 2\pi x$. We wish to construct finite difference approximations to $u(x)$ as on p. 53 of Strang. We will use MATLAB for this.
   (a) Download the M-file trid.m from the class website. This is a fast tridiagonal solver. Use it to find solutions to the system of finite difference equations for $n = 9, 19, 39$.
   (b) Evaluate the accuracy of your solutions by taking the norm of the difference of the solution vector and the vector of exact solutions values at the nodes.

7. P3.1.9, p.142 **Strang.**

Although you may discuss the problems with your classmates, all work should be your own. Cooperative efforts are not permitted.