

1. P3.2.14, p.152 **Strang**.
2. P3.3.24, p.165 **Strang**.
3. P3.4.6, p.180 **Strang**.
4. Let \mathbf{P}^2 be the vector space of all polynomials of degree ≤ 2 equipped with the inner product

$$\langle p, q \rangle = p(0)q(0) + p(1)q(1) + p(2)q(2).$$

- (a) Find $\|1\|$, $\|t\|$, $\|t^2\|$.
 - (b) Find the orthogonal projection of $p_1(t) = t$ on $p_0(t) = 1$.
 - (c) Apply the Gram-Schmidt process to $\{1, t, t^2\}$ to construct an orthogonal basis of \mathbf{P}^2 .
5. P3.4.27, p.182 **Strang**.
 6. (MATLAB) In a paper dealing with the efficiency of energy utilization of the larvae of the Modest Sphinx moth (*Pachysphinx modesta*), L. Schroeder used the following data to determine a quadratic least-squares log-log relation between W , the live weight of the larvae in grams, and R , the oxygen consumption of the larvae in ml/hr. The form of the relation was

$$\log R = a + b \log W + c(\log W)^2,$$

where a, b and c are constants determined by the data. Here $\log W$ is the common logarithm (MATLAB: `log10`).

| W | R | W | R | W | R |
|-------|-------|-------|------|-------|-------|
| 0.017 | 0.154 | 0.783 | 1.47 | 0.111 | 0.357 |
| 0.233 | 0.537 | 2.75 | 1.84 | 1.11 | 0.531 |
| 1.32 | 1.15 | 3.02 | 2.01 | 1.69 | 1.44 |
| 4.29 | 3.40 | 5.45 | 3.52 | 4.83 | 4.66 |

Form the 12×3 data matrix A and find the vector $(a, b, c)^T$ in 4 different ways.

- (a) By using the backslash operator.
 - (b) By forming and solving the normal equations. Note the condition number of the matrix $A^T A$.
 - (c) By using the QR decomposition.
 - (d) By using the Singular-Value Decomposition. All this is quite easy in MATLAB. Plot the graph of R versus W with the values of a, b and c you found. On the same graph plot the data points. Note: if the data is represented as vectors R and W , to plot them do “`plot(R, W, 'o')`”.
7. P3.6.22, p.207 **Strang**. (Read the material on weighted least squares starting on p.203.)

8. Let

$$f(x) = \begin{cases} -1 & -\pi \leq x \leq 0, \\ 1 & 0 < x \leq \pi. \end{cases}$$

- (a) Expand f in a Fourier Series. Since f is an odd function there will only be terms involving $\sin nx$.
- (b) Plot $f(x)$ and the first few partial sums on the same graph. Note: in MATLAB, once you have defined a vector of x -values (for example $x=\text{linspace}(-\pi,\pi,101)$), an easy way to create the function values is $f(x)$ is “ $f(x) = (x > 0) - (x < 0)$ ”.
- (c) How many terms are needed in the sum to get an error $< .15$?