MATH 401 Dr. Wolfe ASSIGNMENT #3 Due June 28, 2004

1. P5.1.18, p.253 **Strang.**

2. P5.3.3, p.272 **Strang.**

3. P5.4.8, p.287 **Strang.**

4. A herd of bison can be modeled by a linear dynamical system. The females can be divided into calves (up to 1 year old), yearlings (1 to 2 years), and adults. Suppose an average of 42 female calves are born each year per 100 adult females. (Only adults produce offspring.) Each year, about 60% of the calves survive, 75% of the yearlings survive, and 95% of the adults survive. For $k \geq 0$,let $\mathbf{x_k} = (c_k, y_k, a_k)^T$, where the entries in $\mathbf{x_k}$ are the number of females in each life stage at year k.
   (a) Constuct the matrix A such that $\mathbf{x_{k+1}} = A\mathbf{x_k}$ for $k \geq 0$.
   (b) (MATLAB) Show that the bison heard is growing, determine the growth rate after many years, and give the expected number of calves and yearlings present per 100 adults.
   (c) (MATLAB) By experimentation find the smallest birthrate (to the nearest integer) for which the population will grow if all the other parameters remain the same.

5. (MATLAB) "All Gaul is divided into three parts." Call them East Gaul, Central Gaul and West Gaul. Each year 5% of the residents of East Gaul move to Certral Gaul and 5% move to West Gaul. Of the residents of Central Gaul 15% move to East Gaul and 10% move to West Gaul. And of the residents of West Gaul, 10% move to East Gaul and 5% move to Central Gaul. What percentage of the population resides in each of the three regions after a long period of time ?

6. (MATLAB) Solve $\mathbf{x}' = A\mathbf{x},\ \mathbf{x}(0) = \mathbf{x_0}$ where

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}, \quad \mathbf{x_0} = \begin{bmatrix} 6 \\ -4 \\ 4 \end{bmatrix}.$$

7. Solve $\mathbf{x}' = C\mathbf{x},\ \mathbf{x}(0) = \mathbf{x_0}$ where

$$C = \begin{pmatrix} 5 & -5 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{x_0} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}.$$

8. A matrix A is <u>nilpotent</u> if $A^k = 0$ for some positve integer k. Show that if A is nilpotent its only eigenvalue is zero. (Note: The converse of this is also true.)

9.
   (a) Suppose A is a diagonalizable matrix, all of whose eigenvalues have absolute value less than 1. Show that $A^n \to 0$ as $n \to \infty$.

(b) Let A be as in part (a). Show that $I - A$ is invertible and

$$(I - A)^{-1} = I + A + A^2 + A^3 + \cdots.$$

(c) Let

$$A = \begin{pmatrix} .2 & -.2 \\ .3 & .7 \end{pmatrix}.$$

Use MATLAB to sum the series and verify the result of part (b). For this set $I = \text{eye}(2), S = I, S = I + S * A$. Now by using the "up-arrow" repeat the last command until you have convergence. Compare with $(I - A)^{-1}$.