MATH 410 Dr. Wolfe ASSIGNMENT #1 Due September 13, 2006

1. The Dedikind Cut Axiom is as follows: Let L and U be nonempty subsets of $\mathbf{R}$ such that $L \cup U = \mathbf{R}$, $L \cap U = \phi$ and if $x \in L$, $y \in U$ then $x < y$. Then either L has a maximum or U has a minimum.

(a) Show that the Least Upper Bound Property implies the Dedikind Cut Axiom.

(b) (Extra Credit) Show that the Dedekind Cut Axiom implies the Least Upper Bound Property.

2. Ex. 7, Sec. 1.1 *Cooper.*

3. Ex. 2, 5, Sec. 1.2 *Cooper.*

4. Ex. 1, 2, Sec. 1.3 *Cooper.*

5. Ex. 2, 4, Sec. 1.4 *Cooper.* .

6.

(a) Using the fact that the square of a real number is nonnegative, prove that for any numbers a and b,

$$ab \leq \frac{1}{2}(a^2 + b^2).$$

(b) Use the result of part (a) to prove that if $a \geq 0$ and $b \geq 0$, then

$$\sqrt{ab} \leq \frac{1}{2}(a + b).$$