MATH 410 Dr. Wolfe ASSIGNMENT #2 Due September 20, 2006

1.For each of the following statements, determine whether it is true or false and justify your answer:
(a) If the sequence $\{a_n^2\}$ converges, then the sequence $\{a_n\}$ also converges.
(b) If the sequence $\{a_n + b_n\}$ coverges, then the sequences $\{a_n\}$ and $\{b_n\}$ also converge.
(c) If the sequences $\{a_n + b_n\}$ and $\{a_n\}$ converge, then the sequence $\{b_n\}$ also converges.
(d) If the sequence $\{|a_n|\}$ converges , then the sequence $\{a_n\}$ also converges.

2. Ex. 1, 5, 8, Sec. 2.1, *Cooper.*

3.Let $\{a_n\}$ be a sequence such that

$$|a_{n+1} - L| \leq \rho |a_n - L| \text{ for all } n > N_0$$

where $0 < \rho < 1$. Prove that $\{a_n\}$ converges to L.

4. The Fibonacci sequence is given by $a_1 = 1$, $a_2 = 1$, $a_{n+2} = a_n + a_{n+1}$ for $n \geq 1$. Let $u_k = a_{k+1}/a_k$. It is known that $\lim_{k\to\infty} u_k = L$.
(a) Find L.
(b) (Extra Credit) Prove $\lim_{k\to\infty} u_k = L$. Hint: You need the result of problem 3.

5. Ex.9, Sec. 2.1, *Cooper.*