MATH 410 Dr. Wolfe ASSIGNMENT #5 Due October 18 , 2006

1. Ex. 3, 4, 7, Sec. 4.1, *Cooper.*

2. Ex. 2, 3, 6, 10, Sec. 4.2, *Cooper.*

3. Suppose that the function $f : \mathbf{R} \to \mathbf{R}$ has the property that

$$-x^2 \leq f(x) \leq x^2 \text{ for all } x.$$

Prove that f is differentiable at $x = 0$ and that $f'(0) = 0$.

4. For real numbers a and b, define

$$g(x) = \begin{cases} 3x^2 & \text{if } x \leq 1 \\ a + bx & \text{if } x > 1. \end{cases}$$

For what values of a and b is the function $g : \mathbf{R} \to \mathbf{R}$ differentiable at $x = 1$?

5. For each of the following statements, determine whether it is true of false and justify your answer.

(a) If the differentiable function $f : \mathbf{R} \to \mathbf{R}$ is strictly increasing,then $f'(x) > 0$ for all x.

(b) If the differentiable function $f : \mathbf{R} \to \mathbf{R}$ is monotonically increasing, then $f'(x) \geq 0$ for all x.

(c) If the function $f : \mathbf{R} \to \mathbf{R}$ is differentiable and

$$f(x) \leq f(0) \text{ for all } x \in [-1, 1],$$

then $f'(0) = 0$.

(d) If the function $f : \mathbf{R} \to \mathbf{R}$ is differentiable and

$$f(x) \leq f(1) \text{ for all } x \in [-1, 1],$$

then $f'(1) = 0$.