MATH 410 Dr. Wolfe ASSIGNMENT #6 Due October 25 , 2006

1. Prove or disprove the following statement: $f(x) = x - \sin x$ is strictly increasing on $\mathbf{R}$.

2. Ex. 1, 2, 5, 7, 8, Sec. 4.3-4.4, *Cooper.*

3. Ex. 8, 11, Sec. 5.1, *Cooper.*

4. Let the function $f : \mathbf{R} \to \mathbf{R}$ have the property that there is a positive number c such that $|f(u) - f(v)| \leq c(u - v)^2$ for all $u, v \in \mathbf{R}$. Prove that the function $f : \mathbf{R} \to \mathbf{R}$ is constant.

5. Let the function $f : \mathbf{R} \to \mathbf{R}$ have two derivatives and suppose that

$$f(x) \leq 0 \quad \text{and } f''(x) \geq 0 \quad \text{for all } x.$$

Prove that $f : \mathbf{R} \to \mathbf{R}$ is constant. (*Hint:* observe that $f' : \mathbf{R} \to \mathbf{R}$ is increasing.)