MATH 411 Dr. Wolfe ASSIGNMENT #2 Due October 4, 2004

1. Suppose S_1 and S_2 are arcwise connected subsets of $\mathbf{R^n}$ with $S_1 \cap S_2 \neq \emptyset$.
   (a) Show that $S_1 \cup S_2$ is arcwise connected.
   (b) By drawing a picture in the plane, show that $S_1 \cap S_2$ is not necessarily arcwise connected.

2. Suppose S_1 and S_2 are convex subsets of $\mathbf{R^n}$ with $S_1 \cap S_2 \neq \emptyset$.
   (a) Show that $S_1 \cap S_2$ is convex.
   (b) By drawing a picture in the plane, show that $S_1 \cup S_2$ is not necessarily convex.

3. Let S be a nonempty subset of $\mathbf{R^n}$. Define

$$C_H = \cap \{K \mid S \subset K, K \text{ convex}\}$$

   C_H is called the *convex hull* of S
   (a) Show C_H is nonempty and convex. (See the argument in 2(a).)
   (b) Sketch C_H when $S = \{(x, y) \mid y = \sqrt{1 - x^2}, -1 \leq x \leq 1\}$.

4. We say $F : \mathbf{R^n} \to \mathbf{R^p}$ is a *linear function* if (i) $F(\mathbf{p_1} + \mathbf{p_2}) = F(\mathbf{p_1}) + F\mathbf{p_2})$ for all $\mathbf{p_1}, \mathbf{p_2} \in \mathbf{R^n}$. and (ii) $F(\alpha \mathbf{p}) = \alpha F(\mathbf{p})$ for all $\mathbf{p} \in \mathbf{R^n}, \alpha \in \mathbf{R}$.
   (a) Show that if F is a linear function and is continuous at $\mathbf{p_0} = \mathbf{0}$ then F is uniformly continuous on $\mathbf{R^n}$.
   (b) If F is a linear function and $S \subset \mathbf{R^n}$ is convex, show that $F(S)$ is also convex.
   (c) Find a function with $n = 2, p = 1$ which satisfies (ii) but not (i).

5. Ex.10, p.269, *Fitzpatrick.*

6. Ex.12, p.309, *Fitzpatrick.*

7. Ex.3, p.317, *Fitzpatrick.*

8. Ex.13, p.319, *Fitzpatrick.*