MATH 411 Dr. Wolfe ASSIGNMENT #6 Due December 8, 2004

1. Assume f is continuous and non-negative on $\mathbf{R^2}$. Given any rectangle R prove $\int_R f = 0$ if and only if $f = 0$ on R.
2. Assume f is continuous on $\mathbf{R^2}$. If $\int_R f = 0$ for all rectangles R, show that $f = 0$ on $\mathbf{R^2}$. (Use proof by contradiction.)
3. Assume that the mixed partials f_{xy} and f_{yx} are continuous on an open set $\mathcal{O}$ and $R = \{(x, y) : a \le x \le b, c \le y < d\} \subset \mathcal{O}$.
   (a) Show: $\int_R f_{xy} = \int_R f_{yx} = f(b, d) + f(a, c) - f(a, d) - f(b, c)$.
   (b) Use part (a) and problem 2 to show that $f_{xy} = f_{yx}$ on $\mathcal{O}$.
4. Let $R = \{(x, y) : 0 \le x, y \le 1\}, f(x, y) = x + y$, and P_n represent the partition of R which divides the x and y axes into n equal sub-intervals. Calculate $U(P, f)$ and from this obtain $\int_R f$.
5. Let
$$f(x, y) = \begin{cases} 2xy^3, & x \text{ rational} \\ xy, & x \text{ irrational} \end{cases}$$
and R as in problem 4. Explain why $\int_R f$ does not exist and then calculate $\int_0^1 (\int_0^1 f(x, y)\, dy)\, dx$.
6. Let $R = \{(x, y) : 0 \le x, y \le 1\}$ and assume f is increasing in x for each fixed y and increasing in y for each fixed x on R. Show that f is integrable over R. (Let P_n be as in problem 4 and examine $U(f, P_n) - L(f, P_n)$ carefully.)
7. If f is integrable over a rectangle R and $\int_R f > 0$ show that the set $\{(x, y) \in R : f(x, y) > 0\}$ has non-empty interior.
8. Ex. 2, p. 459, *Fitzpatrick.*
9. Ex. 7, p. 471, *Fitzpatrick.*
10. Ex. 8, p. 471, *Fitzpatrick.*