MATH 411 Dr. Wolfe SAMPLE FIRST HOUR EXAM Fall 2010

1.

(a) Give an example of an infinite colection of open sets $U_k, k \in N$ in R such that $\cap_k U_k$ is not open.

(b) Give an example of an infinite collection of closed sets $F_k, k \in N$ in R such that $\cup_k F_k$ is not closed.

2. Let A be a subset of $\mathbf{R^n}$. The *characteristic function* of the set A is the function $f : \mathbf{R^n} \to \mathbf{R}$ defined by

$$f(\mathbf{u}) = \begin{cases} 1 & \mathbf{u} \in A \\ 0 & \mathbf{u} \notin A \end{cases}$$

Prove that this characteristic function is continuous at each interior point of A and at each exterior point of A but fails to be continuous at each boundary point of A.

3. Let $f(x, y) = |x^2y^2 + x^3y^3|^{1/4}$.

(a) Prove that f is continuous at $(0, 0)$.

(b) Compute the partial derivatives of f at $(0, 0)$.

(c) Show directly from the definition that f is not differentiable at $(0, 0)$.

4. Let

$$f(x, y) = \frac{1}{x} + \frac{1}{y} + xy.$$

(a) Show that $(1, 1)$ is a critical point of f, i.e. the derivative vector of f vanishes there.

(b) Decide whether $(1, 1)$ is a local maximizer, a local minimizer or a saddle point for f.