MATH 411 Dr. Wolfe ASSIGNMENT #2 Due October 4, 2010

Recall that a set $A \subset \mathbf{R^n}$ is *convex* if whenever $\mathbf{u}$, $\mathbf{v} \in A$ then the line segment $\{t\mathbf{u} + (1-t)\mathbf{v} \,|\, 0 \leq t \leq 1\} \subset A$.

1. Suppose S_1 and S_2 are convex subsets of $\mathbf{R^n}$ with $S_1 \cap S_2 \neq \emptyset$.
(a) Show that $S_1 \cap S_2$ is convex.
(b) By drawing a picture in the plane, show that $S_1 \cup S_2$ is not necessarily convex.

2. Let S be a nonempty subset of $\mathbf{R^n}$. Define

$$C_H = \cap\{K \mid S \subset K, K \text{ convex}\}$$

C_H is called the *convex hull* of S
(a) Show C_H is nonempty and convex. (See the argument in 1(a).)
(b) Sketch C_H when $S = \{(x, y) \mid y = \sqrt{1-x^2}, -1 \leq x \leq 1\}$.

3. We say $F : \mathbf{R^n} \to \mathbf{R^p}$ is a *linear function* if (i) $F(\mathbf{p_1} + \mathbf{p_2}) = F(\mathbf{p_1}) + F\mathbf{p_2})$ for all $\mathbf{p_1}, \mathbf{p_2} \in \mathbf{R^n}$. and (ii) $F(\alpha\mathbf{p}) = \alpha F(\mathbf{p})$ for all $\mathbf{p} \in \mathbf{R^n}, \alpha \in \mathbf{R}$.
(a) Show that if F is a linear function and is continuous at $\mathbf{p_0} = \mathbf{0}$ then F is uniformly continuous on $\mathbf{R^n}$.
(b) If F is a linear function and $S \subset \mathbf{R^n}$ is convex, show that $F(S)$ is also convex.
(c) Find a function with $n = 2, p = 1$ which satisfies (ii) but not (i).

4. Ex. 9, p. 309, *Fitzpatrick.*

5. Ex. 12, p. 353, *Fitzpatrick.*

6. Consider $f : \mathbf{R^2} \to \mathbf{R}$ given by $f(x, y) = 0$ whenever $y \leq 0$ or $y \geq x^2$ and $f(x, y) = 1$ for $0 < y < x^2$. For any fixed value of α show that $f(x, \alpha x) \to 0$ as $x \to 0$ (f has limit zero along any line). However, determine a sequence $(x_k, y_k) \to (0, 0)$ where $f(x_k, y_k) \to 1$ (so that f is discontinuous at the origin).

7. Ex. 7, p.371, *Fitzpatrick.*

8. Ex 9, p.371, *Fitzpatrick.* .