MATH 411 Dr. Wolfe ASSIGNMENT #3 Due October 18, 2010

1. Consider $f : \mathbf{R^n} \to \mathbf{R}$ such that $f(\mathbf{p_1} + \mathbf{p_2}) = f(\mathbf{p_1})f(\mathbf{p_2})$ for all $\mathbf{p_1}, \mathbf{p_2} \in \mathbf{R^n}$.
(a) Show that either $f(\mathbf{0}) = 0$ or $f(\mathbf{0}) = 1$.
(b) Give a (non-constant) example of such an $f$ with $f(0) = 1$ when $n = 1$.
(c) Assume $f(\mathbf{0}) = 1$. Show that $f$ is continuous on $\mathbf{R^n}$ if and only if $f$ is continuous at $\mathbf{0}$.

2. Consider $f(x, y) = |x - y|$. Determine all directions $\mathbf{p}$ for which $\frac{\partial f}{\partial \mathbf{p}}(1, 1)$ exists.

3. Consider $f$ defined by

$$f(x_1, \ldots, x_n) = \begin{cases} x_1 x_2 \cdots x_n/(x_1^2 + x_2^2 + \cdots x_n^2), & (x_1, \ldots, x_n) \neq (0, \ldots, 0) \\ 0, & (x_1, \ldots, x_n) = (0, \ldots, 0) \end{cases}$$

Determine all values of $n$ for which $f$ is differentiable at the origin.

4. (Rolle's Theorem) Suppose $f : \mathbf{R^n} \to \mathbf{R}$ is a differentiable function such that $f(\mathbf{p}) = 0$ if $\|\mathbf{p}\| = 1$. Show that there is a $\mathbf{q}$ with $\|\mathbf{q}\| < 1$ such that $\mathbf{D}f(\mathbf{q}) = \mathbf{0}$. (For $n = 1$, this is Rolle's theorem. Examine the proof in this case.)

5. Ex.15, p.379, *Fitzpatrick.*

6. Ex.17, p.379, *Fitzpatrick.*

7. Ex.3, p.392, *Fitzpatrick.*

8. Ex.11, p.393, *Fitzpatrick.*